

# Aptitude Made Simple

## Why to choose?

Unique as compared to other books

Real Time examples for each topic

Simple to understand for anyone with Mathematical or Non mathematical background

Not just solutions oriented but gives logical tips & tricks for solving problems

## Who can be benefited?

Grade 1 to 10 Maths and Olympiad exams

All Government Exams

All Banking Exams

Aptitude Test for Placement

All Entrance exams for Graduation/ Master level

All Competitive Exams

## Topics Covered

- Divisibility Test
- Power/Exponent/Index
- Number Series
- Average
- Problems on Numbers
- Ages
- Time & Work
- Pipes & Cisterns
- Partnership
- Time & Distance
- Problem on Trains
- Boats & Streams
- Simple Interest
- Compound Interest
- Clocks
- Calendar
- Multiplication Shortcut Tricks
- Mental Calculation Tricks
- Direct & Inverse Proportion
- Power Cycle (Unit's Place Digit)
- Fractions
- HCF-LCM
- Mixture of Allegations
- Percentage
- Probability
- Profit & Loss
- Ratio & Proportion
- Rounding Off
- Trigonometry

## How to get?

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**Amit Manekar**

## About the author

**Amit Manekar** is the Founder of **Aptitude Made Simple**, where initiative is to share simplified Maths, Aptitude content all over the world. He has completed BE Computers from KBT COE(Pune University). He has pursued CDAC from sunbeam Pune. He has 11+ years of experience in IT industry with multiple MNCs as Automation Consultant.

- Achieved 45th All India Rank in CDAC Entrance Exam
- Creating Simplified content from year 2021 and sharing it across with aspirants
- Trained 100+ students on Aptitude for multiple Competitive exams, placements, Olympiad exams etc.
- Visited as a guest lecturer in Diploma and Engineering Colleges, Sainik school entrance exam coaching centre
- Simplified notes of Aptitude Made Simple are referred by Multiple teachers while teaching in their institutes.
- Resolving doubts of students with simplified solutions across multiple platforms.

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1. *The way you teach is so important to us because there is all clarity on all content what you share with me. Also I will buy your all courses after my examination.*
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3. *Aptitude Made Simple preparing people for employment, shaping their careers, fostering personal growth, and aiding in educational and professional decision making. Amit sir is doing a Noble job... Thank you!!*
4. *Dear Sir, Big fan of your work bro.*
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8. *Sir, you are doing excellent work for making mathematics easy, simple, to understand ordinary students and creating interest to learn mathematics. My best wishes to you for your activities.*



**9. Greetings from Australia.**

*What an amazing job, you have done by creating the content of Aptitude Made Simple. As my both the sons are preparing for RMS entrance exam, your simple explanation in the above book is helping all of us to understand the basics of all operations. I would like to express my deepest sense of gratitude towards the contribution in this field. I also would like to thank you for your undivided attention for students studying all over the world would definitely be of great help. As a name suggest Simple: It is actually very SIMPLE. Thanks to bottom of my Heart!!!*

- 10.** *My deepest gratitude for the invaluable mentorship you have provided to me all these times. Your guidance, wisdom and encouragement have been instrumental in my personality and in my professional growth. I am truly thankful to you for the singular dedication, patience and time spent on me. I must mention in particular your insight and providing of short tricks, which were quite useful to me while attempting to move through the fast calculation mode.*
- 11.** *Your channel has been great help to me. Especially, Time and Work, Time and distance, pipes and cisterns and Boats and Streams, these topics before I was planning to skip but once I read on your channel in one day I understood them well.*
- 12.** *Whoever is studying for pre-cat contact this person, he will prepare you for section A in maximum 5 days. He has helped me.*
- 13.** *I am part of your group, Aptitude Made Simple and I must say your tricks are indeed amazing!!! Thank you so much for providing such valuable content that our education system lacks in curriculum and that too for free!! Keep up the Good Work!*
- 14.** *At the time of CCAT I had not studied anything and just your pdf's thoroughly just one day before the exam and I got 325 rank because of your content only. It is highly recommended. Amit ji. Thank you so much for your support. I have been teaching my students with the help of your notes. It's helping a lot. Thanks*
- 15.** *You are a good motivation and good teacher. Your simplified content helped me to get placed in MNC like TCS.*

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## Numbers – Divisibility Test

### What is Divisibility Test?

When you look at any number whether it is 2 digit or 4 digit or 10 digit number, without actually dividing that number with 2, 3, 4 (any number) if you can tell that number is divisible by 2, 3, 4.... number completely that is nothing but divisibility Test.

Let us take sample example to understand it and we will go through divisibility tests for 2 to 11.

### Sample example

Write your mobile on piece of paper. I have written mine below:

**9096754428** Now looking at number tell if this number is completely divisible by 9 or not?

- You will now try to divide complete number by 9 and check what remainder is. If remainder is 0 means completely divisible. Correct?
- Now just do simple thing: Add all digits of given number
- Here:  $9 + 0 + 9 + 6 + 7 + 5 + 4 + 4 + 2 + 8 = 54$
- Now check if 54 is completely divisible by 9 or not.
- Yes 54 is divisible by 9

**So, The number 9096754428 is also completely divisible by 9**

(This is nothing but divisibility test of 9 and each number has different divisible test)

### Why Divisibility Test?

It simplifies your calculation. It also changes the way we look at numbers. When you do regular practice of divisibility test (2 to 11 numbers) and you look at any number you will think whether it is divisible by 2, 3, 4 ...11 etc and will make your calculations quick and short.

You will see lot of benefits of this in all different topics while solving in it.

**Benefits of Divisibility Tests:**

|   |                |
|---|----------------|
| HCF and LCM   | Simplification |
| Work and Time   | Average        |
| Pipes and Cistem  | Series         |
| Quick calculation and any topic where quick calculations are required |                |

**Divisibility Test for 2:**

**Rightmost digit should be divisible by 2.**

All even numbers are completely divisible by 2.

In simple words, if rightmost last digit of any number is 0 or 2 or 4 or 6 or 8 then that number is completely divisible by 2.

Look at below numbers and observe:

| Number | Observation [right most digit]                        | Divisible by 2 or not        |
|--------|---|------------------------------|
| 35     | Rightmost Last digit is 5 and 5 is not divisible by 2 | Number is not divisible by 2 |
| 70     | Rightmost Last digit is 0 and 0 is divisible by 2     | Number is divisible by 2     |
| 269    | Rightmost Last digit is 9 and 9 is not divisible by 2 | Number is not divisible by 2 |
| 1002   | Rightmost Last digit is 2 and 2 is divisible by 2     | Number is divisible by 2     |

**Problem 1:**

Number 3789\* is completely divisible by 2. What number could be there in place of \* ?

**Solution :**

**As per divisibility test for 2, any number whose rightmost digit is 0 or 2 or 4 or 6 or 8 is completely divisible by 2.**

So in place of \* there could be below values:

2, 4, 6, 8, 0

**Answer is 2, 4, 6, 8, 0**



**Problem 2 :**

Which of the following numbers are completely divisible by 2 ?

1236665, 888886, 9999995, 9876543

**Solution :**

**As per divisibility test for 2, any number whose rightmost digit is divisible by 2 is completely divisible by 2.**

So let us look at **right most number** and decide if number is divisible by 2 or not.

| Number  | Observation   | Divisible by 2 or not           |
|---------|---|---------------------------------|
| 1236665 | Rightmost Last digit is 5 and 5 is not divisible by 2 | Number is not divisible by 2    |
| 888886  | Rightmost Last digit is 6 and 6 is divisible by 2     | <b>Number is divisible by 2</b> |
| 9999995 | Rightmost Last digit is 5 and 5 is not divisible by 2 | Number is not divisible by 2    |
| 9876543 | Rightmost Last digit is 3 and 3 is not divisible by 2 | Number is not divisible by 2    |

**Answer is 888886 is completely divisible by 2**

**Divisibility Test for 3 :**

**If sum of all digits of the number is divisible by 3 then that number is completely divisible by 3.**

Look at below numbers and observe:

| Number | Observation [sum of all digits]  | Divisible by 3 or not           |
|--------|--|---------------------------------|
| 123    | Sum of all digits ( $1 + 2 + 3 = 6$ )<br><b>6 is divisible by 3</b>  | <b>Number is divisible by 3</b> |
| 396    | Sum of all digits ( $3 + 9 + 6 = 18$ )<br>18 -> Sum of digits ( $1 + 8 = 9$ )<br><b>9 is divisible by 3</b>                | <b>Number is divisible by 3</b> |
| 775    | Sum of all digits ( $7 + 7 + 5 = 19$ )<br>19 -> Sum of digits ( $1 + 9 = 10$ )<br>10 is not divisible by 3                 | Number is not divisible by 3    |
| 8679   | Sum of all digits ( $8 + 6 + 7 + 9 = 30$ )<br>30 -> Sum of digits ( $3 + 0 = 3$ )<br><b>3 is completely divisible by 3</b> | <b>Number is divisible by 3</b> |

**Note:**

When you do sum of all digits and you get answer which is big number.  
Then continue doing same process till you get single digit sum

Example: 9999968

Sum of digits ( $9 + 9 + 9 + 9 + 9 + 6 + 8 = 59$ )

So some of students may not be able to conclude by looking at number 59 if it is divisible by 3 or not.

Then repeat sum of digit process till you get single digit number

$59 \rightarrow 5 + 9 = 14$

$14 \rightarrow 1 + 4 = 5$

5 is not divisible by 3.

So number 9999968 is not divisible by 3

**Problem 1:**

Number  $5^*985$  is completely divisible by 3. What number could be there in place of  $*$ ?

**Solution :**

**As per divisibility test for 3, if sum of all digits of number is divisible by 3 then that number is also divisible by 3**

Let us take sum of all digits of number  $5^*985$

Sum =  $5 + 9 + 8 + 5 + * = 27 + *$

Digit sum of 27 =  $2 + 7 = 9$  which is divisible by 3 already

In order to have number  $5^*985$  completely divisible sum has of digits should be also divisible by 3.

As Sum without  $*$  is already divisible by 3, we need to have such number in place of  $*$  which will be divisible by 3 and 3, 6, 9, 0 these numbers are completely divisible by 3.

**So answer is 3, 6, 9, 0**

**Problem 2:**

Number  $5^*699^*6$  is completely divisible by 3. There is same number in place of both  $*$ . What number could be there in place of  $*$ ?

**A)1   B)0   C)7   D)2**

**Solution :**

**As per divisibility test for 3, if sum of all digits of number is divisible by 3 then that number is also divisible by 3**

Let us take sum of all digits of number 5\*985

Sum =  $5 + 6 + 9 + 9 + 6 + * + * = 35 + 2*$ . Sum of 35  $\rightarrow 3 + 5 = 8$

To find number in place of \* we need to make sure

$8 + 2*$  should be completely divisible by 3.

If we put 1 at \*  $8 + (2*1) = 10$  not divisible by 3

If we put 2 at \*  $8 + (2*2) = 12$  divisible by 3 Like this we need to check for 0 to 9 and then conclude

**However if we have options given it is best way to use options to solve such questions.**

| Options              | Observation [ sum of all digits as per divisibility test of 3]   | Answer valid/ Not                 |
|----------------------|--|-----------------------------------|
| A (* = 1)<br>5169916 | Sum of all digits ( $5 + 1 + 6 + 9 + 9 + 1 + 6 = 37$ )<br>$37 \rightarrow$ Sum of digits ( $3 + 7 = 10$ )<br>10 is not divisible by 3  | Option A is not the answer        |
| A (* = 0)<br>5069906 | Sum of all digits ( $5 + 0 + 6 + 9 + 9 + 0 + 6 = 35$ )<br>$35 \rightarrow$ Sum of digits ( $3 + 5 = 8$ )<br>8 is not divisible by 3  | Option B is not the answer        |
| A (* = 7)<br>5769976 | Sum of all digits ( $5 + 7 + 6 + 9 + 9 + 7 + 6 = 49$ )<br>$49 \rightarrow$ Sum of digits ( $4 + 9 = 13$ )<br>$13 \rightarrow$ Sum of digits ( $1 + 3 = 4$ )<br>4 is not divisible by 3 | Option C is not the answer        |
| A (* = 2)<br>5269926 | Sum of all digits ( $5 + 2 + 6 + 9 + 9 + 2 + 6 = 39$ )<br>$39 \rightarrow$ Sum of digits ( $3 + 9 = 12$ ) $\rightarrow 1 + 2 = 3$<br><b>3 is completely divisible by 3</b>             | <b>Option D is correct answer</b> |

**Divisibility Test for 4 :**

If last 2 digits part of number is divisible by 4 then that number is completely divisible by 4.

So look at number and just focus on right side last 2 digit number

| Number  | Observation [ last 2 digits]  | Divisible by 4 or not           |
|---------|---|---------------------------------|
| 1236664 | Rightmost Last 2 digits :<br><b>64</b><br>64 is divisible by 4            | <b>Number is divisible by 4</b> |
| 8888860 | Rightmost Last 2 digits :<br><b>60</b><br>60 is divisible by 4            | <b>Number is divisible by 4</b> |
| 1399999 | Rightmost Last 2 digits :<br><b>99</b><br>99 is not divisible by 4        | Number is not divisible by 4    |
| 9376544 | Rightmost Last 2 digits :<br><b>44</b><br>44 is completely divisible by 4 | <b>Number is divisible by 4</b> |
| 9000    | Rightmost Last 2 digits :<br><b>00</b><br>00 is completely divisible by 4 | <b>Number is divisible by 4</b> |

**Problem 1:**

Number 665998\*0 is completely divisible by 4. What number can be there in place of \*?

**Solution :**

**As per divisibility test for 4, if last 2 digit number is divisible by 4 then that number is divisible by 4**

Let us try to check all combination and then find required number: **Focus last 2 digits only**

| Possibility in place of * | Last 2 digits ( when value put for *) | Possible answer(Yes/No) |
|---------------------------|---------------------------------------|-------------------------|
| <b>0</b>                  | 00 -> Divisible by 4                  | <b>Yes</b>              |
| <b>1</b>                  | 10 -> Not divisible by 4              | <b>No</b>               |

|          |                          |            |
|----------|--------------------------|------------|
| <b>2</b> | 20 -> Divisible by 4     | <b>Yes</b> |
| <b>3</b> | 30 -> Not divisible by 4 | <b>No</b>  |
| <b>4</b> | 40 -> Divisible by 4     | <b>Yes</b> |
| <b>5</b> | 50 -> Not divisible by 4 | <b>No</b>  |
| <b>6</b> | 60 -> Divisible by 4     | <b>Yes</b> |
| <b>7</b> | 70 -> Not divisible by 4 | <b>No</b>  |
| <b>8</b> | 80 -> Divisible by 4     | <b>Yes</b> |
| <b>9</b> | 90 -> Not divisible by 4 | <b>No</b>  |

**Answer is 0, 2, 4, 6, 8**

**Note:**

Most of the time options are given and we can directly refer option to solve this

Let us do next problem with option

**Problem 2:**

Number 6977\*\* is completely divisible by 4. What number could be there in place of \* ?

**A) 7    B) 9    C) 5    D) 4**

**Solution :**

Let us put all options values at place of \* 1 by 1 and check for divisibility test of 4

| <b>Options</b>              | <b>Observation [ Last 2 digits as per divisibility test of 4]</b> | <b>Answer valid/ Not</b>          |
|-----------------------------|---|-----------------------------------|
| <b>A (* = 7)<br/>697777</b> | 77 -> Not Divisible by 4  | Option A is not the answer        |
| <b>B (* = 9)<br/>697799</b> | 99 -> Not Divisible by 4  | Option B is not the answer        |
| <b>C (* = 5)<br/>697755</b> | 55 -> Not Divisible by 4  | Option C is not the answer        |
| <b>D (* = 4)<br/>697744</b> | <b>44 -&gt; Divisible by 4</b>                                    | <b>Option D is correct answer</b> |



**Divisibility Test for 5 :**

**If Rightmost digit number is 0 or 5 then number is completely divisible by 5.**

Look at below numbers and observe

| Number | Observation [right most digit 0 Or 5] | Divisible by 5 or not        |
|--------|---------------------------------------|------------------------------|
| 35     | Rightmost Last digit is 5             | Number is divisible by 5     |
| 70     | Rightmost Last digit is 0             | Number is divisible by 5     |
| 269    | Rightmost Last digit is 9             | Number is not divisible by 5 |
| 1000   | Rightmost Last digit is 0             | Number is divisible by 5     |

**Problem 1:**

Number 3789\* is completely divisible by 5. What number could be there in place of \* ?

**Solution :**

**As per divisibility test for 5, any number whose rightmost digit is 0 or 5 is completely divisible by 5**

So in place of \* there could be below values: 0 or 5

**Answer is 0, 5**

**Problem 2:**

Which of the following numbers are completely divisible by 5 ?

1236665, 888886, 9999995, 9876543

**Solution :**

**As per divisibility test for 5, any number whose rightmost digit is 0 or 5 is divisible by 5.**

So let us look at **right most number** and decide if number is divisible by 5 or not.

| Number  | Observation [Rightmost digit 0 or 5] | Divisible by 5 or not           |
|---------|--------------------------------------|---------------------------------|
| 1236665 | Rightmost Last digit is 5            | <b>Number is divisible by 5</b> |
| 888886  | Rightmost Last digit is 6            | Number is not divisible by 5    |
| 9999995 | Rightmost Last digit is 5            | <b>Number is divisible by 5</b> |
| 9876543 | Rightmost Last digit is 3            | Number is not divisible by 5    |

**Answer is 1236665, 9999995 are completely divisible by 5**

### Divisibility Test for 6 :

If number is divisible by 2 and 3 then it is completely divisible by 6

| Number | Divisible by 2              | Divisible by 3   | Divisible by 6 or not     |
|--------|-----------------------------|--|---------------------------|
| 360    | Rightmost digit 0 -><br>Yes | Sum of digit<br>$3 + 6 = 9$<br>9 is divisible by 3   | Number is divisible 6     |
| 375    | Rightmost digit 5 -><br>No  | Not required   | Number is not divisible 6 |
| 4800   | Rightmost digit 0 -><br>Yes | Sum of digit<br>$4 + 8 = 12$<br>$12 \rightarrow 1 + 2 = 3$<br>3 is divisible by 3          | Number is divisible 6     |
| 2436   | Rightmost digit 6 -><br>Yes | Sum of digits<br>$2 + 4 + 3 + 6 = 15$<br>$15 \rightarrow 1 + 5 = 6$<br>6 is divisible by 3 | Number is divisible 6     |

### Divisibility Test for 8 :

If last 3 digits part of number is divisible by 8 then that number is completely divisible by 8.

So look at number and just focus on right side last 3 digit number

| Number          | Observation [ last 3 digits]                                     | Divisible by 8 or not        |
|-----------------|--|------------------------------|
| 1236 <b>064</b> | Rightmost Last 3 digits : <b>064</b><br>64 is divisible by 8     | Number is divisible by 8     |
| 8888 <b>160</b> | Rightmost Last 3 digits : <b>160</b><br>60 is divisible by 8     | Number is divisible by 8     |
| 1399 <b>999</b> | Rightmost Last 3 digits : <b>999</b><br>99 is not divisible by 8 | Number is not divisible by 8 |

|                 |  |                                 |
|-----------------|--|---------------------------------|
| 9376 <b>144</b> | Rightmost Last 3 digits :<br><b>144</b><br>144 is completely divisible<br>by 4 | <b>Number is divisible by 8</b> |
| <b>9000</b>     | Rightmost Last 2 digits :<br><b>00</b><br>00 is completely divisible<br>by 4   | <b>Number is divisible by 8</b> |

**Problem 1:**

Number 665998\*0 is completely divisible by 8. What number can be there in place of \*?

**Solution :**

**As per divisibility test for 8, if last 3 digit number is divisible by 8 then that number is divisible by 8**

Let us try to check all combination and then find required number: **Focus last 3 digits only**

| Possibility in place of * | Last 3 digits ( when value put for *) | Possible answer(Yes/No) |
|---------------------------|---------------------------------------|-------------------------|
| <b>0</b>                  | 800 -> Divisible by 8                 | <b>Yes</b>              |
| <b>1</b>                  | 810 -> Not divisible by 8             | <b>No</b>               |
| <b>2</b>                  | 820 -> Not divisible by 8             | <b>No</b>               |
| <b>3</b>                  | 830 -> Not divisible by 8             | <b>No</b>               |
| <b>4</b>                  | 840 -> Divisible by 8                 | <b>Yes</b>              |
| <b>5</b>                  | 850 -> Not divisible by 8             | <b>No</b>               |
| <b>6</b>                  | 860 -> Not divisible by 8             | <b>No</b>               |
| <b>7</b>                  | 870 -> Not divisible by 8             | <b>No</b>               |
| <b>8</b>                  | 880 -> Divisible by 8                 | <b>Yes</b>              |
| <b>9</b>                  | 890 -> Not divisible by 8             | <b>No</b>               |

**Answer is 0, 4, 8**

**Note:**

Most of the time options are given and we can directly refer option to solve this

Let us do next problem with option:

**Problem 2:**

Number 69777\* is completely divisible by 8. What number could be there in place of \* ?

A) 5    B) 4    C) 6    D) 0

**Solution :**

Let us put all options values at place of \* 1 by 1 and check for divisibility test of 4

| Options             | Observation [ Last 3 digits as per divisibility test of 8] | Answer valid/ Not          |
|---------------------|--|----------------------------|
| A (* = 5)<br>697775 | 775 -> Not Divisible by 8                                  | Option A is not the answer |
| B (* = 4)<br>697774 | 774 -> Not Divisible by 8                                  | Option B is not the answer |
| C (* = 6)<br>697776 | 776 -> Divisible by 8                                      | Option C is correct answer |
| D (* = 0)<br>697770 | 770 -> Not divisible by 8                                  | Option D is not the answer |

Once we get answer we don't need to check next options [just written for reference and understanding]

**Divisibility Test for 9 :**

If sum of all digits of the number is divisible by 9 then that number is completely divisible by 9.

Look at below numbers and observe

| Number | Observation [sum of all digits]   | Divisible by 9 or not        |
|--------|---|------------------------------|
| 123    | Sum of all digits (1 + 2 + 3 = 6)<br>6 is not divisible by 9  | Number is divisible by 9     |
| 396    | Sum of all digits (3 + 9 + 6 = 18)<br>18 -> Sum of digits (1 + 8 = 9)<br>9 is completely divisible by 9     | Number is divisible by 9     |
| 775    | Sum of all digits (7 + 7 + 5 = 19)<br>19 -> Sum of digits (1 + 9 = 10)<br>10 is not divisible by 9          | Number is not divisible by 9 |
| 8676   | Sum of all digits (8 + 6 + 7 + 6 = 27)<br>27 -> Sum of digits (2 + 7 = 9)<br>9 is completely divisible by 9 | Number is divisible by 9     |

**Problem 1:**

Number  $5^*985$  is completely divisible by 9. What number could be there in place of  $*$ ?

**Solution :**

**As per divisibility test for 9, if sum of all digits of number is divisible by 9 then that number is also divisible by 9**

Let us take sum of all digits of number  $5^*985$

$$\text{Sum} = 5 + 9 + 8 + 5 + * = 27 + *$$

Digit sum of 27 =  $2 + 7 = 9$  which is divisible by 9 already

In order to have number  $5^*985$  completely divisible sum has of digits should be also divisible by 9

As Sum without  $*$  is already divisible by 9, we need to have such number in place of  $*$  which will be divisible by 9

9, 0 these numbers are completely divisible by 9

**So Answer is 9, 0**

**Problem 2:**

Number  $5^*699^*6$  is completely divisible by 9. There is same number in place of both  $*$ . What number could be there in place of  $*$ ?

A) 1   B) 0   C) 7   D) 2

**Solution :**

**As per divisibility test for 9, if sum of all digits of number is divisible by 9 then that number is also divisible by 9**

Let us take sum of all digits of number  $5^*699^*6$

$$\text{Sum} = 5 + 6 + 9 + 9 + 6 + * + * = 35 + 2*$$

$$\text{Sum of } 35 \rightarrow 3 + 5 = 8$$

To find number in place of  $*$  we need to make sure

$8 + 2*$  should be completely divisible by 9.

If we put 1 at  $*$   $8 + (2 \times 1) = 10$  not divisible by 9

If we put 2 at  $*$   $8 + (2 \times 2) = 12$  not divisible by 9

Like this we need to check for 0 to 9 and then conclude.

**However if we have options given it is best way to use options to solve such questions.**



| Options              | Observation [ sum of all digits as per divisibility test of 9]  | Answer valid/ Not                     |
|----------------------|---|---------------------------------------|
| A (* = 1)<br>5169916 | Sum of all digits ( 5 + 1 + 6 + 9 + 9 + 1 + 6 = 37)<br>37 -> Sum of digits (3 + 7 = 10)<br>10 is not divisible by 9   | Option A is not the answer            |
| A (* = 0)<br>5069906 | Sum of all digits (5 + 0 + 6 + 9 + 9 + 0 + 6 = 35)<br>35 -> Sum of digits (3 + 5 = 8)<br>8 is not divisible by 9  | Option B is not the answer            |
| A (* = 5)<br>5569956 | Sum of all digits (5 + 5 + 6 + 9 + 9 + 5 + 6 = 45)<br>45 -> Sum of digits (4 + 5 = 9)<br><b>9 is divisible by 9</b>   | <b>Option C is the correct answer</b> |
| A (* = 2)<br>5269926 | Sum of all digits (5 + 2 + 6 + 9 + 9 + 2 + 6 = 39)<br>39 -> Sum of digits (3 + 9 = 12)<br>12 -> Sum of digits (1 + 2 = 3)<br><b>3 is not divisible by 9</b> | Option D is not the answer            |

**Divisibility Test for 10:**

If Rightmost digit of number is 0 then number is completely divisible by 10.

Look at below numbers and observe

| Number | Observation [right most digit 0] | Divisible by 10 or not        |
|--------|----------------------------------|-------------------------------|
| 3000   | Rightmost Last digit is 0        | Number is divisible by 10     |
| 70     | Rightmost Last digit is 0        | Number is divisible by 10     |
| 269    | Rightmost Last digit is 9        | Number is not divisible by 10 |
| 1000   | Rightmost Last digit is 0        | Number is divisible by 10     |

**Problem 1:**

Number 3788869\* is completely divisible by 10 What number could be there in place of \* ?

**Solution :**

As per divisibility test for 10, any number whose rightmost digit is 0 is completely divisible by 10

So in place of \* there could be below values:

0

**Answer is 0**

### Problem 2:

Which of the following numbers are completely divisible by 10 ?

12366650, 888886, 99999950, 9876543

**Solution :**

**As per divisibility test for 10, any number whose rightmost digit is 0 is completely divisible by 10.**

So let us look at **right most number** and decide if number is divisible by 10 or not.

| Number   | Observation [Rightmost digit 0] | Divisible by 10 or not               |
|----------|---------------------------------|--------------------------------------|
| 1236665  | Rightmost Last digit is 5       | Number is not divisible by 10        |
| 8888860  | Rightmost Last digit is 0       | Number is completely divisible by 10 |
| 99999950 | Rightmost Last digit is 0       | Number is completely divisible by 10 |
| 9876543  | Rightmost Last digit is 3       | Number is not divisible by 10        |

**Answer is 8888860, 99999950 is completely divisible by 10**

### Divisibility Test for 11:

Calculate sum of digits at odd places (1, 3, 5, ... from left)

Calculate sum of digits at even places (2, 4, 6, ... from left)

Now get difference of both:

Sum of odd places digits – Sum of even places digits.

Sample Number: 278354

|           |            |           |            |           |            |
|-----------|------------|-----------|------------|-----------|------------|
| 2         | 7          | 8         | 3          | 5         | 4          |
| Odd place | Even place | Odd place | Even place | Odd place | Even place |

Odd places sum =  $2 + 8 + 5 = 15$

Even places sum =  $7 + 3 + 4 = 14$

Difference = Odd place sum – Even places sum

$$= 15 - 14 = 1$$

As 1 is not divisible by 11 number 278354 is also not divisible by 11

If this difference is divisible by 11 then number is divisible by 11

[respective sign of difference]

Let us look at some examples to understand:

| Number | Sum of odd places digits | Sum of even places digits | Difference [ Odd places sum – even place sum] | Divisible by 11 or not |
|--------|--------------------------|---------------------------|---|------------------------|
| 451    | $4 + 1 = 5$              | 5                         | $5 - 5 = 0$<br>0 is divisible by 11           | Divisible by 11        |
| 12133  | $1 + 1 + 3 = 5$          | $2 + 3 = 5$               | $5 - 5 = 0$<br>0 is divisible by 11           | Divisible by 11        |
| 3916   | $3 + 1 = 4$              | $9 + 6 = 15$              | $4 - 15 = -11$<br>-11 is divisible by 11      | Divisible by 11        |
| 7586   | $7 + 8 = 15$             | $5 + 6 = 11$              | $15 - 11 = 4$<br>4 is not divisible by 11     | Not divisible by 11    |

### Summary of Divisibility Tests

| Number | Divisibility test   |
|--------|---|
| 2      | Rightmost digit number should be divisible by 2 [ 0, 2, 4, 6, 8 at rightmost] |
| 3      | Sum of all digits of the number should be divisible by 3                      |
| 4      | Last 2 digit number should be divisible by 4                                  |
| 5      | Rightmost digit number should be 0 or 5                                       |
| 6      | Number should be divisible by 2 and 3   |
| 8      | Last 3 digit number should be divisible by 8                                  |
| 9      | Sum of all digits of the number should be divisible by 9                      |
| 10     | Rightmost digit number should be 0  |
| 11     | Sum of odd places digit – sum of even places digit should be divisible by 11  |

### Power Cycle ( $x^n$ – Unit Place Digit)

Each of the aptitude question exam paper contains 2 Or more questions on identifying unit place digit for some number(x) to the power of some other number (n)

Let us look at 1 of the example :

What will be unit's place digit for  $77^{123}$  ?

- Do you really feel that we are supposed to multiply 77 number 123 times during exam?
- **Obviously Not!** We need to work with the way so that without doing that much multiplication we should be capable of answering

**Power cycle will help us to solve this problem.**

Let us take real life example to understand power cycle

Let us assume **Sun** as number for which index n is to be calculated.

Sun follows below 2 steps:

Step 1: Sunrise

Step 2: Sunset

So let us look at power and respective values for Sun to the power n

|                           |         |
|---------------------------|---------|
| <b><math>Sun^1</math></b> | Sunrise |
| <b><math>Sun^2</math></b> | Sunset  |
| <b><math>Sun^3</math></b> | Sunrise |
| <b><math>Sun^4</math></b> | Sunset  |
| <b><math>Sun^5</math></b> | Sunrise |

Looking at above table we can see that Sun follow pattern Sunrise, Sunset  
**(Sunrise, Sunset) is Power cycle for Sun**

We will identify power cycles for all numbers 0 to 9 and you would be able to solve any problem asked in exam easily and post practice orally as well  
 If you can remember power cycle values that would be great however even if you are unable to remember it, we will look at technique to **calculate power cycle of number in less than 1 minute during exam** as well and you can solve problem.

**Power Cycles for all Numbers (Focus on Unit place digit only)**

**Number 0 :**

Let us calculate values for  $0^1$  to  $0^5$

| Power of 0 | Value |
|------------|-------|
| $0^1$      | 0     |
| $0^2$      | 0     |
| $0^3$      | 0     |
| $0^4$      | 0     |
| $0^5$      | 0     |

So as you can observe

Value of  $0^1$  to  $0^5$  is 0 only. So Unit place digit is 0 for any power of 0

**Power Cycle for 0 : (0)**

**Problem 1**

What will be unit's place digit for  $250^{123}$  ?

**Solution :**

Look at the unit place of number 250

Unit's place digit is 0.

**Power Cycle of 0 : (0)**

**Answer is unit's place digit for  $250^{123}$  will be 0**

**Problem 2**

What will be unit's place digit for  $670^{4123}$  ?

**Solution :**

Look at the unit place of number 670

Unit's place digit is 0.

**Power Cycle of 0 : (0)**

**Answer is unit's place digit for  $670^{4123}$  will be 0**



**Number 1 :**

Let us calculate values for  $1^1$  to  $1^5$

| Power of 1 | Value |
|------------|-------|
| $1^1$      | 1     |
| $1^2$      | 1     |
| $1^3$      | 1     |
| $1^4$      | 1     |
| $1^5$      | 1     |

So as you can observe

Value of  $1^1$  to  $1^5$  is 1 only. So Unit place digit is 1 for any power of 1

**Power Cycle for 1 : (1)****Problem 1**

What will be unit's place digit for  $121^{53}$  ?

**Solution :**

Look at the unit place of number 121

Unit's place digit is 1

**Power Cycle of 1 : (1)**

**Answer is unit's place digit for  $121^{53}$  will be 1**

**Problem 2**

What will be unit's place digit for  $791^{5643}$  ?

**Solution :**

Look at the unit place of number 791.

Unit's place digit is 1

**Power Cycle of 1 : (1)**

**Answer is unit's place digit for  $791^{5643}$  will be 1**

**Number 2 :**

Let us calculate values for  $2^1$  to  $2^5$

| Power of 2 | Value |
|------------|-------|
| $2^1$      | 2     |
| $2^2$      | 4     |
| $2^3$      | 8     |
| $2^4$      | 16    |
| $2^5$      | 32    |

So as you can observe

**Unit place digit of  $2^1$  to  $2^5$  is in order 2, 4, 8, 6 and it will keep repeating as 2, 4, 8, 6**

**Power Cycle for 2 : (2, 4, 8, 6)**

**Problem 1**

What will be unit's place digit for  $2^{33}$  ?

**Solution :**

Look at the unit place of number 2

Unit's place digit is 2.

**Power Cycle of 2 : (2, 4, 8, 6)**

There are total 4 values which keep repeating always for power of 2.

Now look at index which is to be identified 33

As 4 numbers keep on repeating for power cycle of 2, we need to divide 33 by 4 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{33}{4}, \text{Quotient} = 8 \text{ and } \text{Remainder} = 1$$

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 2                |
| 2         | 4                |
| 3         | 8                |
| 0         | 6                |

You don't need to remember this table you just need to make sure as you know pattern of power cycle you have to reach till index number

Like in this case:

To reach 33 and you have size of 4

4, 8, 12 . 32 so 32<sup>nd</sup> index would be last number in power cycle that is 6

33<sup>rd</sup> index would have 1<sup>st</sup> number in power cycle that is 2

**Answer is unit's place digit for  $2^{33}$  will be 2**

### Problem 2

What will be unit's place digit for  $1222^{438}$

**Solution :**

Look at the unit place of number 1222.

Unit's place digit is 2.

**Power Cycle of 2 : (2, 4, 8, 6)**

There are total 4 values which keep repeating always for power of 2.

Now look at index which is to be identified 438

As 4 numbers keep on repeating for power cycle of 2, we need to divide 438 by 4 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{438}{4}, \text{ Quotient} = 109 \text{ and } \text{Remainder} = 2$$

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 2                |
| 2         | 4                |
| 3         | 8                |
| 0         | 6                |

**Answer is unit's place digit for  $1222^{438}$  will be 4**

**Number 3 :**

Let us calculate values for  $3^1$  to  $3^5$

| Power of 3 | Value |
|------------|-------|
| $3^1$      | 3     |
| $3^2$      | 9     |
| $3^3$      | 27    |
| $3^4$      | 81    |
| $3^5$      | 243   |

So as you can observe

**Unit place digit of  $3^1$  to  $3^5$  is in order 3, 9, 7, 1**

**and it will keep repeating as 3, 9, 7, 1**

**Power Cycle for 3 : (3, 9, 7, 1)**

**Problem 1**

What will be unit's place digit for  $3^{36}$  ?

**Solution :**

Look at the unit place of number 3

Unit's place digit is 3.

**Power Cycle of 3 : (3, 9, 7, 1)**

There are total 4 values which keep repeating always for power of 3.

Now look at index which is to be identified 36

As 4 numbers keep on repeating for power cycle of 3, we need to divide 36 by 4 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{36}{4}, \text{Quotient} = 9 \text{ and } \text{Remainder} = 0$$

Whenever remainder is 0 it is last digit in power cycle

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 3                |
| 2         | 9                |
| 3         | 7                |
| 0         | 1                |

**Answer is unit's place digit for  $3^{36}$  will be 1**

**Problem 2**

What will be unit's place digit for  $123^{498}$  ? ?

**Solution :**

Look at the unit place of number 123.

Unit's place digit is 3.

**Power Cycle of 3 : (3, 9, 7, 1)**

There are total 4 values which keep repeating always for power of 3.

Now look at index which is to be identified 498

As 4 numbers keep on repeating for power cycle of 3. we need to divide 498 by 4 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{498}{4}, \text{Quotient} = 124 \text{ and } \text{Remainder} = 2$$

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 3                |
| 2         | 9                |
| 3         | 7                |
| 0         | 1                |

**Answer is unit's place digit for  $123^{498}$  will be 9**

**Number 4 :**

Let us calculate values for  $4^1$  to  $4^5$

| Power of 4 | Value |
|------------|-------|
| $4^1$      | 4     |
| $4^2$      | 16    |
| $4^3$      | 64    |
| $4^4$      | 256   |
| $4^5$      | 1024  |

So as you can observe

**Unit place digit of  $4^1$  to  $4^5$  is in order 4, 6 and it will keep repeating as 4, 6**



**Power Cycle for 4 : (4, 6)****Problem 1**

What will be unit's place digit for  $4^{360}$  ?

**Solution :**

Look at the unit place of number 4.

Unit's place digit is 4.

**Power Cycle of 4 : (4, 6)**

There are total 2 values which keep repeating always for power of 4.

Now look at index which is to be identified 360

As 2 numbers keep on repeating for power cycle of 4, we need to divide 360 by 2 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{360}{2}, \text{Quotient} = 180 \text{ and } \text{Remainder} = 0$$

Whenever remainder is 0 it is last digit in power cycle

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 4                |
| 0         | 6                |

**Answer is unit's place digit for  $4^{360}$  will be 6**

**Problem 2**

What will be unit's place digit for  $1234^{6987}$  ?

**Solution :**

Look at the unit place of number 1234

Unit's place digit is 4.

**Power Cycle of 4 : (4, 6)**

There are total 2 values which keep repeating always for power of 4.

Now look at index which is to be identified 6987

As 2 numbers keep on repeating for power cycle of 3, we need to divide 6987 by 2 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{6987}{2}, \text{Quotient} = 3493 \text{ and Remainder} = 1$$

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 4                |
| 0         | 6                |

**Answer is unit's place digit for  $1234^{6987}$  will be 4**

### **Number 5:**

**Let us calculate values for  $5^1$  to  $5^5$**

| Power of 1 | Value |
|------------|-------|
| $5^1$      | 5     |
| $5^2$      | 25    |
| $5^3$      | 125   |
| $5^4$      | 625   |
| $5^5$      | 3125  |

So as you can observe

**Unit place digit of  $5^1$  to  $5^5$  is 5 only So Unit place digit is 5 for any power of 5**

**Power Cycle for 5 : (5)**

#### **Problem 1**

What will be unit's place digit for  $5^{12}$

**Solution :**

Look at the unit place of number 5

Unit's place digit is 5.

**Power Cycle of 5 : (5)**

**Answer is unit's place digit for  $5^{12}$  will be 5**

#### **Problem 2**

What will be unit's place digit for  $25^{56}$  ?

**Solution :**

Look at the unit place of number 25.

Unit's place digit is 5.

**Power Cycle of 5 : (5)**

**Answer is unit's place digit for  $25^{56}$  will be 5**

### **Number 6:**

**Let us calculate values for  $6^1$  to  $6^5$**

| Power of 6 | Value |
|------------|-------|
| $6^1$      | 6     |
| $6^2$      | 36    |
| $6^3$      | 216   |
| $6^4$      | 1296  |
| $6^5$      | 7776  |

So as you can observe,

**Unit place digit of  $6^1$  to  $6^5$  is 6 only So Unit place digit is 6 for any power of 6**

**Power Cycle for 6 : (6)**

### **Problem 1**

What will be unit's place digit for  $56^{142}$  ?

**Solution :**

Look at the unit place of number 56.

Unit's place digit is 6.

**Power Cycle of 6 : (6)**

**Answer is unit's place digit for  $56^{142}$  will be 6**

### **Problem 2**

What will be unit's place digit for  $286^{56}$  ?

**Solution :**

Look at the unit place of number 286.

Unit's place digit is 6.

**Power Cycle of 6 : (6)**

**Answer is unit's place digit for  $286^{56}$  will be 6**

**Number 7 :**

Let us calculate values for  $7^1$  to  $7^5$

| Power of 7 | Value |
|------------|-------|
| $7^1$      | 7     |
| $7^2$      | 49    |
| $7^3$      | 343   |
| $7^4$      | 2401  |
| $7^5$      | 16807 |

So as you can observe

**Unit place digit of  $7^1$  to  $7^5$  is in order 7, 9, 3, 1**

**and it will keep repeating as 7, 9, 3, 1**

**Power Cycle for 7 : (7, 9, 3, 1)**

**Problem 1**

What will be unit's place digit for  $7^{77}$  ?

**Solution :**

Look at the unit place of number 7

Unit's place digit is 7.

**Power Cycle of 7 : (7, 9, 3, 1)**

There are total 4 values which keep repeating always for power of 7

Now look at index which is to be identified 77

As 4 numbers keep on repeating for power cycle of 7, we need to divide 77 by 4 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{77}{4}, \text{Quotient} = 19 \text{ and } \text{Remainder} = 1$$

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 7                |
| 2         | 9                |
| 3         | 3                |
| 0         | 1                |

**Answer is unit's place digit for  $7^{77}$  will be 7**

**Problem 2**

What will be unit's place digit for  $1237^{496}$  ?

**Solution :**

Look at the unit place of number 1237

Unit's place digit is 7.

**Power Cycle of 7 : (7, 9, 3, 1)**

There are total 4 values which keep repeating always for power of 7

Now look at index which is to be identified 496

As 4 numbers keep on repeating for power cycle of 7, we need to divide 496 by 4 and identify remainder of it so that we can understand what can be unit place number.

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{496}{4}, \text{Quotient} = 124 \text{ and } \text{Remainder} = 0$$

Whenever remainder is 0 that is last number of power cycle

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 7                |
| 2         | 9                |
| 3         | 3                |
| 0         | 1                |

**Answer is unit's place digit for  $1237^{496}$  will be 1**

**Number 8 :**

Let us calculate values for  $8^1$  to  $8^5$

| Power of 8 | Value |
|------------|-------|
| $8^1$      | 8     |
| $8^2$      | 64    |
| $8^3$      | 512   |
| $8^4$      | 4096  |
| $8^5$      | 32768 |

So as you can observe

**Unit place digit of  $8^1$  to  $8^5$  is in order 8, 4, 2, 6 and it will keep repeating as 8, 4, 2, 6**

**Power Cycle for 8 : (8, 4, 2, 6)**

**Problem 1**

What will be unit's place digit for  $8^{67}$  ?

**Solution :**

Look at the unit place of number 8

Unit's place digit is 8.

**Power Cycle of 7 : (8, 4, 2, 6)**

There are total 4 values which keep repeating always for power of 8.

Now look at index which is to be identified 67

As 4 numbers keep on repeating for power cycle of 8, we need to divide 67 by 4 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{67}{4}, \text{Quotient} = 14 \text{ and Remainder} = 3$$

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 8                |
| 2         | 4                |
| 3         | 2                |
| 0         | 6                |

**Answer is unit's place digit for  $8^{67}$  will be 2**

**Problem 2**

What will be unit's place digit for  $128^{6802}$  ?

**Solution :**

Look at the unit place of number 128.

Unit's place digit is 8.

**Power Cycle of 8 : (8, 4, 2, 6)**

There are total 4 values which keep repeating always for power of 8.

Now look at index which is to be identified 6802

As 4 numbers keep on repeating for power cycle of 8. we need to divide 6802 by 4 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{6802}{4}, \text{ Quotient} = 1700 \text{ and Remainder} = 2$$

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 8                |
| 2         | 4                |
| 3         | 2                |
| 0         | 6                |

**Answer is unit's place digit for  $128^{6802}$  will be 4**

### **Number 9 :**

**Let us calculate values for  $9^1$  to  $9^5$**

| Power of 9 | Value |
|------------|-------|
| $9^1$      | 9     |
| $9^2$      | 81    |
| $9^3$      | 729   |
| $9^4$      | 6651  |
| $9^5$      | 59859 |

So as you can observe

**Unit place digit of  $9^1$  to  $9^5$  is in order 9 1 and it will keep repeating as 9, 1**

**Power Cycle for 9 : (9, 1)**

### **Problem 1**

What will be unit's place digit for  $9^{99}$  ?

**Solution :**

Look at the unit place of number 9

Unit's place digit is **9**.

**Power Cycle of 9 : (9, 1)**

There are total 2 values which keep repeating always for power of 9

Now look at index which is to be identified 99

As 2 numbers keep on repeating for power cycle of 9, we need to divide 99 by 2 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{99}{2}, \text{Quotient} = 49 \text{ and } \text{Remainder} = 1$$

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 9                |
| 0         | 1                |

**Answer is unit's place digit for  $9^{99}$  will be 9**

**Problem 2**

What will be unit's place digit for  $999^{1000}$  ?

**Solution :**

Look at the unit place of number 999

Unit's place digit is **9**.

**Power Cycle of 9 : (9, 1)**

There are total 2 values which keep repeating always for power of 9

Now look at index which is to be identified 1000

As 2 numbers keep on repeating for power cycle of 9, we need to divide 1000 by 2 and identify remainder of it so that we can understand what can be unit place number

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{1000}{2}, \text{Quotient} = 500 \text{ and } \text{Remainder} = 0$$

| Remainder | Unit Place digit |
|-----------|------------------|
| 1         | 9                |
| 0         | 1                |

Whenever remainder is 0 it is last digit in power cycle.

**Answer is unit's place digit for  $999^{1000}$  will be 1**



**Summary of Power Cycle (Unit place Digit)**

| Number | Power Cycle | Size of power Cycle |
|--------|-------------|---------------------|
| 0      | 0           | 1                   |
| 1      | 1           | 1                   |
| 2      | 2, 4, 8, 6  | 4                   |
| 3      | 3, 9, 7, 1  | 4                   |
| 4      | 4, 6        | 2                   |
| 5      | 5           | 1                   |
| 6      | 6           | 1                   |
| 7      | 7, 9, 3, 1  | 4                   |
| 8      | 8, 4, 2, 6  | 4                   |
| 9      | 9, 1        | 2                   |

### Number Series

Various competitive examinations ask questions regularly based on number series, therefore candidates must possess the required knowledge. Number series can be quickly solved and is a biggest time saver!!!

Number series refers to a sequence of numbers following some pattern. In our day to day life whenever we come across any problem we think whether we or any of our friend or family has already faced the problem

Depending on those previous experience (**pattern**) and what actions we did to solve it so we can use same thing to solve our current problems

#### What is pattern and what is series?

Let us assume you live on 6th floor and you are going by stairs. So you will go from 0th floor to 1st floor 1st floor to 2nd floor and subsequently from 5th floor to 6th floor

So each time we are going 1 floor up.

**Each of these floor is pattern and our journey from Ground floor to 6th floor is Series.**

Pre-requisites for solving series problems quickly and orally

1. Tables 1 to 30
2. Squares 1 to 30
3. Cubes 1 to 10
4. Prime numbers 2 to 100

We will be looking at approaches to solve number series problems

|   |                   |
|---|-------------------|
| Addition (+)  | Squares ( $n^2$ ) |
| Subtraction (-)                                       | Cubes ( $n^3$ )   |
| Multiplication (*)                                    | Odd/Even number   |
| Division (/)  | Prime Number      |
| Miscellaneous [Combination of 2 or more any of these] |                   |

**Approach 1: Addition****Problem 1 :** 7, 11, 15, 19, 23, ?**Solution :**

Look at the series and try to identify what is pattern?

Series is in increasing (ascending) order

How much added in 7 to get 11?  $\rightarrow 4$ How much added in 11 to get 15?  $\rightarrow 4$ How much added in 15 to get 19?  $\rightarrow 4$ How much added in 19 to get 23?  $\rightarrow 4$ **Pattern:** 4 is getting added in number to get next number

|   |    |    |    |    |   |
|---|----|----|----|----|---|
| 7 | 11 | 15 | 19 | 23 | ? |
|   | +4 | +4 | +4 | +4 |   |

To find number in place of ? we need to add 4 to 23

 $23+4=27$ .**Answer is 27.****Problem 2 :** 8, 12, 17, 23, 30, ?**Solution :**

Look at the series and try to identify what is pattern?

All numbers are increasing order

How much added in 8 to get 12  $\rightarrow 4$ How much added in 12 to get 17  $\rightarrow 5$ How much added in 17 to get 23  $\rightarrow 6$ How much added in 23 to get 30  $\rightarrow 7$ **Pattern** 4,5,6,7, ..are getting added in number to get next number

|   |    |    |    |    |   |
|---|----|----|----|----|---|
| 8 | 12 | 17 | 23 | 30 | ? |
|   | +4 | +5 | +6 | +7 |   |

To find number in place of ? we need to add 8 in 30

 $30+8 = 38$ **Answer is 38**

**Approach 2: Subtraction****Problem 1 :** 45, 35, 25, 15, ?**Solution :**

Look at the series and try to identify what is pattern?

It is in decreasing (descending) order.

How much to be subtracted from 45 to get 35  $\rightarrow$  10How much to be subtracted from 35 to get 25  $\rightarrow$  10How much to be subtracted from 25 to get 15  $\rightarrow$  10**Pattern** 10 is getting subtracted from number to get next number

|   |    |    |    |   |
|---|----|----|----|---|
| 45  | 35 | 25 | 15 | ? |
| -10                      -10                      -10 |    |    |    |   |

To find number in place of ? we need to subtract 10 from 15

$$15 - 10 = 5$$

**Answer is 5****Problem 2 :** 72, 63, 54, 45, ?**Solution :**

Look at the series and try to identify what is pattern?

It is in decreasing (descending) order.

If look at the numbers carefully and you know table of 9, you can easily figure it out that all numbers are from table of 9

How much to be subtracted from 72 to get 63  $\rightarrow$  9How much to be subtracted from 63 to get 54  $\rightarrow$  9How much to be subtracted from 54 to get 45  $\rightarrow$  9**Pattern** 9 is getting subtracted from number to get next number

|  |    |    |    |   |
|--|----|----|----|---|
| 72   | 63 | 54 | 45 | ? |
| -9                      -9                      -9 |    |    |    |   |

To find number in place of ? we need to subtract 9 from 45

$$45 - 9 = 36$$

**Answer is 36.**

**Approach 3: Multiplication**

In this approach there is common multiplication factor from 1st term to 2nd term and 2nd term to 3rd term in series till last term.

**Problem 1:** 4, 8, 16, 32, ?

**Solution :**

Look at the series and try to identify what is pattern?

With which number 4 is to be multiplied to get 8-> 2

With which number 8 is to be multiplied to get 16-> 2

With which number 16 is to be multiplied to get 32-> 2

**Pattern:** Number is multiplied by 2 to get next number.

|    |   |    |    |   |
|----|---|----|----|---|
| 4  | 8 | 16 | 32 | ? |
| *2 |   | *2 | *2 |   |

To find number in place of ? we need to multiply 32 by 2.

$$32 * 2 = 64$$

**Answer is 64.**

**Problem 2:** 8, 12, 18, 27, ?

**Solution :**

Look at the series and try to identify what is pattern?

As it is increasing try to see if addition pattern exists.

|    |    |    |    |   |
|----|----|----|----|---|
| 8  | 12 | 18 | 27 | ? |
| +4 |    | +6 | +9 |   |

Not able to see any pattern of addition and So we will check for multiplication

So there is multiplication factor 1.5

Note here multiplication factor can be decimal as well

**Pattern:** Number is multiplied by 1.5 to get next number

|      |    |      |      |   |
|------|----|------|------|---|
| 8    | 12 | 18   | 27   | ? |
| *1.5 |    | *1.5 | *1.5 |   |

To find number in place of ? we need to multiply 27 by 1.5

Multiplication by 1.5 means adding half of number to it.

$$27 + (27/2) = 27 + 13.5 = 40.5$$

**Answer is 40.5**

#### Approach 4: Division

**Problem 1:** 200, 100, 50, 25, ?

**Solution :**

Look at the series and try to identify what is pattern?

With which number 200 is to be divided to get 100-> 2

With which number 100 is to be divided to get 50-> 2

With which number 50 is to be divided to get 25-> 2

**Pattern** Number is divided by 2 to get next number

|     |     |    |    |   |
|-----|-----|----|----|---|
| 200 | 100 | 50 | 25 | ? |
| /2  |     | /2 | /2 |   |

To find number in place of ? we need to divide 25 by 2

$$25 / 2 = 12.5$$

**Answer is 12.5**

**Problem 2:** 270, 90, 30, ?

**Solution :**

Look at the series and try to identify what is pattern?

With which number 270 is to be divided to get 90-> 3

With which number 90 is to be divided to get 30-> 3

**Pattern** Number is divided by 3 to get next number

|     |    |    |    |
|-----|----|----|----|
| 270 | 90 | 30 | ?  |
| /3  |    | /3 | /3 |

To find number in place of ? we need to divide 30 by 3

**Answer is 10**

**Approach 5: Squares**

If you have squares learned from 1 to 30 it would help you and speed up your calculation in each aptitude topic.

**Problem 1:** 9, 16, 25, 36, 49, ?

**Solution :**

Look at the series and try to identify what is pattern?

You can see 1 number is added and its square is calculated

**Pattern:** 1 is added to current number and its square is calculated to get next number

|       |           |           |           |           |   |
|-------|-----------|-----------|-----------|-----------|---|
| 9     | 16        | 25        | 36        | 49        | ? |
| $3^2$ | $(3+1)^2$ | $(4+1)^2$ | $(5+1)^2$ | $(6+1)^2$ |   |
| $3^2$ | $4^2$     | $5^2$     | $6^2$     | $7^2$     |   |

To find number in place of ? we need to add 1 in 7 and calculate square of it.

$$(7+1)^2 = 8^2 = 64$$

**Answer is 64**

**Problem 2:** 121, 100, 81, 64, ?

**Solution :**

Look at the series and try to identify what is pattern?

**Pattern:** 1 is reduced to current number and its square is calculated to get next number

|        |            |            |           |   |
|--------|------------|------------|-----------|---|
| 121    | 100        | 81         | 64        | ? |
| $11^2$ | $(11-1)^2$ | $(10-1)^2$ | $(9-1)^2$ |   |
| $11^2$ | $10^2$     | $9^2$      | $8^2$     |   |

To find number in place of ? we need to reduce 1 from 8 and calculate square of it.

$$(8-1)^2 = 7^2 = 49$$

**Answer is 49**

**Approach 6: Cubes**

If you have cubes learned from 1 to 10 it would help you and speed up your calculation in each aptitude topic.

**Problem 1:** 1, 8, 27, 64, ?

**Solution :**

Look at the series and try to identify what is pattern?

**Pattern:** 1 is added to current number and its cube is calculated to get next number

|       |           |           |           |   |
|-------|-----------|-----------|-----------|---|
| 1     | 8         | 27        | 64        | ? |
| $1^3$ | $(1+1)^3$ | $(2+1)^3$ | $(3+1)^3$ |   |
| $1^3$ | $2^3$     | $3^3$     | $4^3$     |   |

To find number in place of ? we need to add 1 to 4 and calculate cube of it

$$(4+1)^3 = 5^3 = 125$$

**Answer is 125**

**Problem 2:** 125, 64, 27, 8, ?

**Solution :**

Look at the series and try to identify what is pattern?

**Pattern:** 1 is reduced from current number and its cube is calculated to get next number

|       |           |           |           |   |
|-------|-----------|-----------|-----------|---|
| 125   | 64        | 27        | 8         | ? |
| $5^3$ | $(5-1)^3$ | $(4-1)^3$ | $(3-1)^3$ |   |
| $5^3$ | $4^3$     | $3^3$     | $2^3$     |   |

To find number in place of ? we need to subtract 1 from 2 and calculate cube of it

$$(2-1)^3 = 1^3 = 1$$

**Answer is 1**



**Approach7: Prime number****Problem 1:** 7, 13, 19, 29, ?**Solution :**

Looking at all terms we can figure out that all are prime number

**Pattern:** After current number consecutive 1<sup>st</sup> prime number is ignored and 2<sup>nd</sup> prime number is taken as next number in series

|                 |    |    |    |           |   |
|-----------------|----|----|----|-----------|---|
| Prime number(n) | 7  | 13 | 19 | 29        | ? |
| n+1 Prime no    | 11 | 17 | 23 | 31        |   |
| n+2 Prime no    | 13 | 19 | 29 | <b>37</b> |   |

To find number in place of ? we need to find 2<sup>nd</sup> consecutive prime number after 291<sup>st</sup> Prime number after 29 = 312<sup>nd</sup> Prime number after 29 = 37**Answer is 37****Problem 2:** 61, 59, 53, 47, ?**Solution :**

Looking at all terms we can figure out that all are prime number

**Pattern:** After current number previous consecutive 1<sup>st</sup> prime number is taken as next number in series

|                 |    |    |    |    |   |
|-----------------|----|----|----|----|---|
| Prime number(n) | 61 | 59 | 53 | 47 | ? |
| n-1 Prime no    | 59 | 53 | 47 | 43 |   |

To find number in place of ? we need to find 1<sup>st</sup> previous consecutive prime number before 47

Previous prime number of 47 = 43

**Answer is 43**

**Approach8: Odd/Even number****Problem 1 :** 23, 25, 27, 29, 31, ?**Solution :**

Looking at all terms we can figure out that all are odd numbers

This can be solved with approach of addition as well as odd number

Consecutive odd number of 23  $\rightarrow$  25Consecutive odd number of 25  $\rightarrow$  27Consecutive odd number of 27  $\rightarrow$  29Consecutive odd number of 29  $\rightarrow$  31

To find number in place of ? we need to find consecutive odd number after 31

**Answer is 33****Problem 2:** 60, 45, 58, 43, 56, 41, ?**Solution:**

Sometimes it may happen that pattern is not in continuous numbers but in alternate numbers

**Pattern** Even numbers are decreasing on 1<sup>st</sup>, 3<sup>rd</sup> and 5<sup>th</sup> term and odd numbers are decreasing in 2<sup>nd</sup>, 4<sup>th</sup> and 6<sup>th</sup> position

| Even n |    | Even n-1 |    | Even n-2 |    |   |
|--------|----|----------|----|----------|----|---|
| 60     | 45 | 58       | 43 | 56       | 41 | ? |
| Odd n  |    | Odd n-1  |    | Odd n-2  |    |   |

To find number in place of ? we need to find previous consecutive even number before 56

**Answer is 54****Miscellaneous:**

This approach contains combination of 2 or more approaches that we discussed earlier

**Problem 1:** 4, -8, 16, -32, 64, ?**Solution:****Pattern:** Number is multiplied by -2 to get next number

|     |    |     |     |     |      |
|-----|----|-----|-----|-----|------|
| 4   | -8 | 16  | -32 | 64  | -128 |
| *-2 |    | *-2 |     | *-2 |      |

**Answer is -128**

**Problem 2:** 16, 33, 65, 131, 261, ?

**Pattern:** 1<sup>st</sup> number multiplied by 2 and 1 added 2<sup>nd</sup> number multiplied by 2 and 1 is subtracted

**Solution**

|                     |    |                     |     |                     |            |
|---------------------|----|---------------------|-----|---------------------|------------|
| 16                  | 33 | 65                  | 131 | 261                 | <b>523</b> |
| $(16 \times 2) + 1$ |    | $(33 \times 2) - 1$ |     | $(65 \times 2) + 1$ |            |

**Answer is 523**

**Problem 3:** 165, 195, 255, 285, 345, ?

**Pattern** 30 added in 1<sup>st</sup> number to get 2<sup>nd</sup> number and 60 gets added to get 3<sup>rd</sup> number. Subsequently 30 and 60 are added to get next numbers in series

**Solution :**

|     |     |     |     |     |            |
|-----|-----|-----|-----|-----|------------|
| 165 | 195 | 255 | 285 | 345 | <b>375</b> |
| +30 |     | +60 |     | +30 |            |

**Answer is 375**

**Problem 4:**

7, 26, 63, 124, 215, 342 ?

**Pattern** 1 number is added to current number and cube of it is calculated and then 1 is reduced from it.

**Solution :**

|           |               |               |               |               |               |               |
|-----------|---------------|---------------|---------------|---------------|---------------|---------------|
| 7         | 26            | 63            | 124           | 215           | 342           | <b>511</b>    |
| $2^3 - 1$ | $(2+1)^3 - 1$ | $(3+1)^3 - 1$ | $(4+1)^3 - 1$ | $(5+1)^3 - 1$ | $(6+1)^3 - 1$ | $(7+1)^3 - 1$ |
| $2^3 - 1$ | $3^3 - 1$     | $4^3 - 1$     | $5^3 - 1$     | $6^3 - 1$     | $7^3 - 1$     | $8^3 - 1$     |

**Answer is 511**

**Problem 5:** 8, 7 11, 12, 14, 17, 17, 22, ?

**Solution :**

**Pattern** Alternate number series with add 3 and 5 in both sub-series

8, 11, 14, 17, 20

7, 12, 17, 22

|   |   |   |   |    |   |    |   |    |   |    |   |    |   |    |           |
|---|---|---|---|----|---|----|---|----|---|----|---|----|---|----|-----------|
| 8 | + | 3 | = | 11 | + | 3  | = | 14 | + | 3  | = | 17 | + | 3  |           |
| 8 |   | 7 |   | 11 |   | 12 |   | 14 |   | 17 |   | 17 |   | 22 | <b>20</b> |
|   |   | 7 | + | 5  | = | 12 | + | 5  | = | 17 | + | 5  | = | 22 |           |

**Answer is 20**

**Problem 6:** 11, 13, 17, 19, 23, 29, 31, 37, 41, ?

**Solution :**

**Pattern:** If you look at all numbers, all are prime numbers and consecutive prime numbers.

After 41 next prime number -> 43

**Answer is 43**

**Problem 7:** 2, 6, 12, 20, 30, 42 56, ?

**Solution :**

**Pattern:** Number is getting added in each number is in sequence

4, 6, 8, 10, 12, 14, 16

|   |    |    |    |     |     |     |           |
|---|----|----|----|-----|-----|-----|-----------|
| 2 | 6  | 12 | 20 | 30  | 42  | 56  | <b>72</b> |
|   | +4 | +6 | +8 | +10 | +12 | +14 | +16       |

**Answer is 72**

**Problem 8:** 80, 99, 120, 143, ?

**Solution:**

**Pattern:** Number is increased by 1 and its square is calculated and 1 is reduced from it.

|         |             |              |              |              |
|---------|-------------|--------------|--------------|--------------|
| 80      | 99          | 120          | 143          | <b>168</b>   |
| $9^2-1$ | $(9+1)^2-1$ | $(10+1)^2-1$ | $(11+1)^2-1$ | $(12+1)^2-1$ |
| $9^2-1$ | $10^2-1$    | $11^2-1$     | $12^2-1$     | $13^2-1$     |

**Answer is 168**

## Average

Various competitive examinations ask questions regularly based on Average. Average is one of the important aspect which we see in each of our observation.

### Why average?

When you plan to take bike/car you will see how much average that car will be giving, even you go for interview or placement you will be asked for average percentage of graduation degree

When we know average, we get high level idea Basically to make us capable of getting overall idea, average is to be solved

### Important formulae:

$$1) \quad \text{Average} = \frac{\text{Sum of All Numbers}}{\text{Number Count}}$$

- 2) Average speed when we travel from Source place A to B and come back from B to A [Assuming distance will be equal as source and destination A, B]

$$\frac{2xy}{x+y}$$

Where:

x speed from Source(A) to Destination (B)

y speed from Destination(B) to Source (A)

### Problem 1

David obtained 76, 65, 82, 67, 85 marks out of 100 in English, Maths, Physics, Chemistry and Biology. What is average of his marks?

**Solution :**

$$\begin{aligned} \text{Average} &= \frac{\text{Total of All Marks}}{\text{Number Count} \text{ Number of Subjects}} \\ &= \frac{76+65+82+67+85}{5} \\ &= \frac{375}{5} = 75 \end{aligned}$$

**Answer is 75 marks**

**Problem 2**

A Student was asked to find arithmetic mean of numbers 3, 11, 7, 9, 15, 13, 8, 19, 7, 21, 14, x. He find mean to be 12. What should be number in place of x.

**Solution:**

Average is 12 and numbers count is also 12.

$$\text{Average} = \frac{\text{Sum of All Numbers}}{\text{Number Count}}$$

$$12 = \frac{13 + 11 + 7 + 9 + 15 + 13 + 8 + 19 + 7 + 21 + 14 + x}{12}$$

$$12 = \frac{137 + x}{12}$$

$$137 + x = 144$$

$$x = 144 - 137 = 7$$

**Answer is 7**

**Problem 3**

What is Average of 50 natural numbers ?

**Solution**

Sum of 1 to 50 (n natural numbers)

$$\begin{aligned} \text{Sum of 1}^{\text{st}} \text{ n natural numbers} &= \frac{n \cdot (n+1)}{2} \\ &= \frac{50 \cdot (50+1)}{2} \\ &= 25 \cdot 51 = 1275 \end{aligned}$$

$$\begin{aligned} \text{Average} &= \text{total} / \text{number count} \\ &= 1275 / 50 = 25.5 \end{aligned}$$

If you observe properly if you are identifying average of consecutive number then 1st number + last number / 2

So average of 1 to 5

$$1+5 / 2 = 3$$

$$1+50 / 2 = 25.5$$

**Answer is 25.5**

**Problem 4**

Average of weight if A, B, C is 45 kg If average weight of A and B is 40 kg  
Average weight of B and C is 43 kg What is weight of B?

**Solution :**

Average weight of A, B and C is 45

Average = Total/ number of values

$$45 = \text{Total}/3$$

$$\text{Total} = 45 * 3 = 135$$

A and B average weight is 40

$$\text{So } A + B = 40 * 2 = 80$$

B and C average weight is 43

$$\text{So } B + C = 43 * 2 = 86$$

To find value of B we should 1st get value of either C or A

$$A+B = (A+B+C) - C$$

$$80 = 135 - C$$

$$C = 55 \text{ kg}$$

To get value of B we have to solve

$$B+C = 86$$

$$B+55 = 86$$

$$B = 31 \text{ kg}$$

**Answer is 31 kg**

**Problem 5:**

Average of 50 numbers is 30. If 35 and 40 are removed from list what will be average of remaining numbers

**Solution :**

As average of 50 numbers is 30

So total of all numbers would be

$$50 * 30 = 1500$$

As 35 and 40 discarded total would be reduced by

$$35 + 40 = 75$$

$$1500 - 75 = 1425$$

As 2 numbers removed numbers are 48

New average = new total / 48

$$= 1425 / 48 = 29.6$$

**Answer is 29.6**

### **Problem 6**

Average of 11 numbers is 10.9. If average of 1st 6 number is 10.5 and that of last 6 numbers is 11.4 Find out middle number in series.

**Solution :**

$$\text{Average} = \frac{\text{Sum of All Numbers}}{\text{Number Count}}$$

$$10.9 = \frac{\text{Sum of All Numbers}}{11}$$

$$\text{Total of all numbers} = 10.9 * 11 = 119.9$$

1<sup>st</sup> 6 numbers average is 10.5

$$\text{Total of 1<sup>st</sup> 6 numbers} = 10.5 * 6 = 63$$

Last 6 numbers average is 11.4

$$\text{Total of last 6 numbers} = 11.4 * 6 = 68.4$$

Total of 12 numbers = Total of 1<sup>st</sup> 6 numbers + Total of last 6 numbers – Middle number

This is because as you can see middle number is common in both places and added twice.

|                           |   |   |   |   |   |                |   |   |    |    |    |
|---------------------------|---|---|---|---|---|----------------|---|---|----|----|----|
| 1                         | 2 | 3 | 4 | 5 | 6 | 7              | 8 | 9 | 10 | 11 | 12 |
| 1 <sup>st</sup> 6 numbers |   |   |   |   |   |                |   |   |    |    |    |
|                           |   |   |   |   |   | Last 6 numbers |   |   |    |    |    |

$$119.9 = 63 + 68.4 - \text{Middle number}$$

$$\text{Middle number} = 131.4 - 119.9 = 11.5$$

**Answer is 11.5**



**Problem 7**

Average of 36 students in group 14 years. When teacher's age is added to it average increases by 1. What is teacher's age?

**Solution:**

$$\text{Average weight} = \frac{\text{Sum of All Weights}}{\text{Number of Students}}$$

$$\text{Sum of all weights} = 36 * 14 = 504$$

Assume teacher's weight  $x$

Once teacher's age is added average is increased by 1.

New average = 15

New Person count = 37

Student + Teacher weight = New average \* New count

$$504 + x = 15 * 37$$

$$x = 555 - 504 = 51$$

**Answer is Teacher's age is 51 years**

**Problem 8**

In 50 Over ODI match, in 1st 10 overs of cricket game run rate was only 3.2. What should be run rate in remaining overs to reach target of 282?

**Solution :**

In 10 overs run rate was 3.2 runs

$$\text{So total score after 10 overs} = 10 * 3.2 = 32$$

Total target = 282 runs

$$\text{Pending runs after 10 over} = 282 - 32 = 250$$

$$\text{Remaining overs} = 50 - 10 = 40$$

$$\begin{aligned}\text{Required run rate} &= \frac{\text{Remaining Runs}}{\text{Remaining Overs}} \\ &= \frac{250}{40} \\ &= 6.25\end{aligned}$$

**Answer is Required run rate is 6.25**

**Problem 9**

Grocer has sale Rs 6435, Rs 6927, Rs 6855, Rs 7230, Rs 6562 for 5 consecutive months. How much should be in 6th month to have average sale is 6500?

**Solution :**

In order to get average sale for 6 month 6500 Rs

Total of sales of 5 months =  $6500 \times 6 = 39000$  Rs

$$\begin{aligned}\text{Total of initial 5 months} &= 6435 + 6927 + 6855 + 7230 + 6562 \\ &= 34009\end{aligned}$$

$$\begin{aligned}\text{6th Month sales should be} &= \text{Total sales} - 1^{\text{st}} \text{ 5 month sales} \\ &= 39000 - 34009 \\ &= 4991\end{aligned}$$

**Answer is 6<sup>th</sup> month sale should be 4991 Rs**

**Problem 10**

Motorist travel 150 km away at average of 30 km/hr and returns at speed of 50 km/hr

His average speed is?

**Solution:**

Please don't confuse here with distance as it is not required at all

Average speed when we travel from Source place A to B and come back from B to A [Assuming distance will be equal as source and destination A, B]

$$= \frac{2xy}{x+y}$$

Where:

x speed from Source(A) to Destination (B)

y speed from Destination(B) to Source (A)

$$= \frac{2 \times 30 \times 50}{30 + 50}$$

$$= \frac{3000}{80}$$

$$= 37.5$$

**Answer is Average speed is 37.5 km/hr**

### Problem11

A cricketer has certain average of 10 innings in 11th inning he scored 108 runs thereby increasing average by 6 runs. His new average??

**Solution :**

We have last inning's score as 108 and total number of innings 11

Let us assume x is original average

So new average is  $x + 6$

Total of 1<sup>st</sup> 10 innings + 108 = 11 \* New average

$$10 * x + 108 = 11 (x + 6)$$

$$10x + 108 = 11x + 66$$

$$x = 108 - 66 = 42$$

Old average is 42

$$\text{New average} = x + 6 = 42 + 6 = 48$$

**Answer is New average is 48**

### Problem12

Average weight of 8 person is increases by 2.5 kg when new person comes in place of one of them weighs 65kg. What might be weight of new number?

**Solution :**

Average weight of 8 person increased by 2.5 kg.

So Original total weight would increase by  $8 * 2.5 = 20$  Kg

In order to get total increase by 20 Kg, new person's weight should be more than 20 from old person

$$\text{New person weight} = \text{old person weight} + 20$$

$$\text{New person weight} = 65 + 20 = 85$$

**Answer is new person's weight is 85 Kg**

**Problem 13**

Average of 10 number is 7 if each number is multiplied by 12 then average of new number is

**Solution :**

Let us look at small example before we solve this

Average of 1, 2, 3, 4, 5 will be 3

Let us multiply each number by 10

Average of 10, 20, 30, 40, 50 will be 30 [  $3 \times 10 = 30$  as a number multiplied by 10]

So you can observe if you multiply all number by common constant

Original average of 10 numbers is 7

If we multiply all numbers by 12, average would also will increase by 12

So new average =  $7 \times 12 = 84$

**Answer is 84**

**Problem 14**

Average of 35 student is 16 years Average of 21 students is 14 years

What is average age of remaining 14 students?

**Solution :**

35 student age average 16 years.

So total of 35 student ages =  $35 \times 16 = 460$

21 students age average is 14 years

So total of 14 student ages =  $21 \times 14 = 294$

We have to find average of pending 14 students

Total of ages of 35 students = Total of ages of 21 students + Total of ages of 14 students

$460 = 294 + \text{Total of Remaining 14 student ages}$

Total of remaining 14 student ages =  $460 - 294 = 166$

Average of 14 student ages = 
$$\frac{\text{Total of remaining 14 student ages}}{14}$$

$$= \frac{166}{14}$$

$$= 19$$

**Answer is average of remaining 14 students is 19 Years**

**Problems on Numbers**

Various competitive exam nations ask questions regularly based on Problems on Numbers.

Many students face challenges in understanding and solving problems as they are not able to convert problem statements from words into equation of numbers.

Let us take real life example and will try to understand concept of Problem of numbers

**Real life example for Problems on Numbers:**

Assume you have 1000 Rs with you and your friend has some amount of Rs. Total money you and your friend has is 2000 Rs. What is amount your friend have?

You will feel this thing very simple  $1000 + 1000 = 2000$

So your friend has 2000 Rs

We just need to solve similar problems but before solving we just need to convert problem statement given in words to equation.

Here we can say equation as

Money you have + Money friend have = 2000

Assume your friend has x Rs with him/her

$$1000 + x = 2000$$

$$x = 1000$$

So basically, if we are able to convert problem statement into equation we have solved 80% of the problem. We will practice initially for this before solving problem

Assume **original number as x** and we will convert problem statement into equation.

| Statement                                       | Re-presentation n equation |
|---|----------------------------|
| Rec procal of number                            | $1/x$                      |
| Number is doubled                               | $2x$                       |
| Number is tripled                               | $3x$                       |
| Half of the number                              | $x/2$                      |
| $1/5^{\text{th}}$ of the number                 | $x/5$                      |
| Number increased by 20                          | $x + 20$                   |
| 20 added to the number                          | $x + 20$                   |
| 10 subtracted from number                       | $x - 10$                   |
| Number multiplied by 12                         | $12x$                      |
| Number divided by 10                            | $x/10$                     |
| Square of number                                | $x^2$                      |
| 3 consecutive natural numbers                   | $x, x+1, x+2$              |
| 4 consecutive odd numbers                       | $x, x+2, x+4, x+6$         |
| 3 consecutive even numbers                      | $x, x+2, x+4$              |
| Sum of squares of 3 consecutive natural numbers | $x^2 + (x+1)^2 + (x+2)^2$  |

Practice these similar statements and most of the time in numbers problem, you will either see 1 or more number of statement combination.

First step in problem would be convert problem statement into equation and then solve it.

Some p aces it might be easy to substitute answers given in option and check whether our equation satisfies it or not.

We will solve problems now

**Problem 1:**

The sum of rational number and its reciprocal is  $13/6$ . Find the number

**Solution :**

Let us assume number as  $x$

$$\text{Reciprocal of number} = \frac{1}{x}$$

$$x + \frac{1}{x} = \frac{13}{6}$$

$$\frac{x^2 + 1}{x} = \frac{13}{6}$$

$$6x^2 + 6 = 13x$$

$$6x^2 - 13x + 6 = 0$$

**Note** I will create 1 separate document on solving quadratic equations

$$6x^2 - 9x - 4x + 6 = 0$$

$$3x(2x-3) - 2(2x-3) = 0$$

$$(3x-2)(2x-3) = 0$$

So either  $(3x-2) = 0$  or  $(2x-3) = 0$

$$x = \frac{2}{3} \quad \text{or} \quad x = \frac{3}{2}$$

**Answer** is number is  $2/3$  or  $3/2$

**Problem 2:**

The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers?

**Solution :**

Let us assume number as  $x$  and  $y$

Difference of 2 numbers = 11

$$x - y = 11 \quad (\text{Equation 1})$$

$$\frac{1}{5}(x + y) = 9$$

$$x + y = 45 \quad (\text{Equation 2})$$

Adding 2 equations:

$$x - y = 11 \quad (\text{Equation 1})$$

$$x + y = 45 \quad (\text{Equation 2})$$

---

$$2x = 56$$

$$x = 28$$

Put value of  $x$  in equation 1

$$x - y = 11$$

$$28 - y = 11$$

$$y = 17$$

**Answer is the numbers are 28 and 17**

**Problem 3:**

The sum of two numbers is 15 and sum of their squares is 113. Find the numbers?

**Solution :**

Let us assume number as  $x$  and  $y$



Sum of number is 15

$$x + y = 15 \quad (\text{Equation 1})$$

Sum of squares is 113

$$x^2 + y^2 = 113 \quad (\text{Equation 2})$$

We need to represent y in form of x so that we can get equation to solve

$$\text{As } x + y = 15 \text{ means } y = 15 - x$$

Put this value of y in Equation 2

$$x^2 + (15 - x)^2 = 113$$

$$\text{use } (a - b)^2 = a^2 - 2ab + b^2$$

$$x^2 + 225 - 30x + x^2 = 113$$

$$2x^2 - 30x + 112 = 0$$

$$2x^2 - 16x - 14x + 112 = 0$$

$$2x(x - 8) - 14(x - 8) = 0$$

$$(2x - 14)(x - 8) = 0$$

$$\text{So } 2x - 14 = 0 \quad \text{or } x - 8 = 0$$

$$2x = 14 \quad \text{or } x = 8$$

$$x = 7$$

When  $x = 7$  then  $y = 15 - 7 = 8$  and vice versa [you can cross check your answer by putting these values in Equation 1 and Equation 2]

**Answer is numbers are 7 and 8 or 8 and 7**

**Problem 4:**

The average of 4 consecutive even numbers is 27 Find the largest of these numbers.

**Solution :**

As you can see numbers are consecutive even.

Let us assume 1<sup>st</sup> smallest number x so next numbers would be  $x+2$  ,  $x+4$  and  $x+6$

Average of 4 number is 27

So total of number would be  $4 \times 27 = 108$

$$x + (x + 2) + (x + 4) + (x + 6) = 108$$

$$4x + 12 = 108$$

$$4x = 96$$

$$x = 24$$

So smallest number is 24

$$\text{Largest number} = x + 6 = 24 + 6 = 30$$

**Answer is largest number is 30**

### **Problem 5:**

Sum of squares of three consecutive odd number is 2531. Find the numbers.

#### **Solution :**

As you can see numbers are consecutive odd

Let us assume 1<sup>st</sup> smallest number  $x$  so next numbers would be  $x+2$ ,  $x+4$

Sum of squares of these three numbers = 2531

$$x^2 + (x + 2)^2 + (x + 4)^2 = 2531 \quad \text{use } (a + b)^2 = a^2 + 2ab + b^2$$

$$x^2 + x^2 + 4x + 4 + x^2 + 8x + 16 = 2531$$

$$3x^2 + 12x + 20 = 2531$$

$$3x^2 + 12x - 2511 = 0$$

Divide by 3 throughout

$$x^2 + 4x - 837 = 0$$

$$x^2 + 31x - 27x - 837 = 0$$

$$x(x + 31) - 27(x + 31) = 0$$

$$(x + 31)(x - 27) = 0$$

$$x = 27 \text{ or } x = -31$$

Take positive number

$$\text{Smallest number} = x = 27$$

$$\text{Next number} = x + 2 = 27 + 2 = 29$$

$$\text{Last number} = x + 4 = 27 + 4 = 31$$

**Answer is numbers are 27, 29 and 31**

**Problem 6:**

The difference between a number and its three-fifth is 50. What is the number?

**Solution :**

Let us assume number as x

Difference between number and its  $\frac{3}{5}$ <sup>th</sup> is 50

$$x - \frac{3}{5}x = 50$$

$$x \left(1 - \frac{3}{5}\right) = 50$$

$$\frac{2}{5}x = 50$$

$$x = \frac{50 \times 5}{2} = 125$$

**Answer is number is 125**

**Problem 7:**

If one-third of one fourth of a number is 15. Then three-tenth of number is?

**Solution :**

Let us assume number as x

one-third of one fourth of a number = 15

$$\frac{1}{3} * \frac{1}{4} * x = 15$$

$$\frac{x}{12} = 15$$

$$x = 180$$

Three tenth of number

$$= \frac{3}{10} * 180 = 3 * 18 = 54$$

**Answer is three-tenth of number is 54**

**Problem 8:**

A number is doubled and 9 is added. If the resultant is trebled, it becomes 75. What is that number?

**Solution :**

Let us assume number as  $x$

Number is doubled and 9 added  $= 2x + 9$

Resultant is trebled  $= 3 * (2x + 9) = 6x + 27 = 75$

$$6x + 27 = 75$$

$$6x = 48$$

$$x = 8$$

**Answer is number is 8**

**Problem 9:**

When 24 is subtracted from a number, it reduces to its four-seventh. What is the sum of digits of that number?

**Solution :**

Let us assume number as  $x$ .

24 subtracted from number  $= x - 24$

4/7<sup>th</sup> of number  $= \frac{4}{7} X$

$$x - 24 = \frac{4}{7} X$$

$$x - \frac{4}{7} X = 24$$

$$x \left(1 - \frac{4}{7}\right) = 24$$

$$x * \frac{3}{7} = 24$$

$$x = \frac{24 * 7}{3} = 8 * 7 = 56$$

The number is 56

Sum of digits of 56  $= 5 + 6 = 11$

**Answer is sum of digits of number is 11**

**Problem 10:**

Find number which when multiplied by 15 is increased by 196.

**Solution :**

Let us assume number as x.

Number is multiplied by 15  $\cdot$  15x

It is increased by 196

This means  $\cdot$

Number multiplied by 15 – number = 196

$$15x - x = 196$$

$$14x = 196$$

$$x = 14$$

**Answer is number is 14**

**Problem 11:**

A number whose 5<sup>th</sup> part increased by 4 is equal to its fourth part diminished by 10, is?

number which when multiplied by 15 is increased by 196.

**Solution :**

Let us assume number as x.

5<sup>th</sup> part of number:  $\frac{x}{5}$

5<sup>th</sup> part Increased by 4:  $\frac{x}{5} + 4$  -----Equation1

4<sup>th</sup> part of number :  $\frac{x}{4}$

4<sup>th</sup> part of number diminished by 10  $\cdot$   $\frac{x}{4} - 10$  -----Equation2

As per problem statement both equation1 and equation2 are equal

$$\frac{x}{5} + 4 = \frac{x}{4} - 10$$

$$\frac{x}{4} - \frac{x}{5} = 4 + 10$$

$$x \left( \frac{1}{4} - \frac{1}{5} \right) = 14$$

$$x \left( \frac{1}{20} \right) = 14$$

$$x = 14 * 20 = 280$$

**Answer is number is 280**

**Problem 12:**

The sum of two numbers is 25 and the difference is 13. Find their product.

**Solution :**

Let us assume numbers are x and y

Sum of numbers is 25:  $x + y = 25$  -----Equation1

Difference of numbers is 13:  $x - y = 13$ -----Equation2

Add Equation1 and Equation2

$$x + y = 25$$

$$+ x - y = 13$$

-----

$$2x = 38$$

$$x = 19$$

Put value of x in Equation1

$$x + y = 25$$

$$19 + y = 25$$

$$y = 6$$

Numbers are 19 and 6

$$\text{Product of numbers: } 19 * 6 = 114$$

**Answer is product of numbers is 114**

**Problem 13**

Three numbers are in ratio 4 : 5 : 6 and their average is 25. Largest number is?

**Solution :**

Numbers are in proportion 4 : 5 : 6 so largest number would be 3<sup>rd</sup> (with 6 in ratio)

Let us assume common multiple of ratio as x.

So our numbers are 4x : 5x : 6x

Average of number is 25 means their sum is  $25 * 3 = 75$

$$4x + 5x + 6x = 75$$

$$15x = 75$$

$$x = 5 \text{ [Common multiple is 5]}$$

$$\text{Largest number} = 6x = 6 * 5 = 30$$

**Answer is largest number is 30**

**Problem 14**

Find positive integer which when increased by 17 is equal to 60 times the reciprocal of the number.

**Solution :**

Let us assume number as x.

$$\text{Reciprocal of number} = \frac{1}{x}$$

$$\text{Integer increased by 17 : } x + 17 \text{-----Equation 1}$$

$$60 \text{ times of reciprocal : } 60 * \frac{1}{x} = \frac{60}{x} \text{-----Equation 2}$$

As per problem statement Equation1 and Equation2 are equal

$$x + 17 = \frac{60}{x}$$

$$x(x + 17) = 60$$

$$x^2 + 17x - 60 = 0$$

$$x^2 + 20x - 3x - 60 = 0$$

$$x(x + 20) - 3(x + 20) = 0$$

$$(x - 3)(x + 20) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x + 20 = 0$$

$$x = 3 \quad \text{or} \quad x = -20$$

As per problem statement, we need positive integer

So  $x = 3$

**Answer is number is 3**

**Problem 15**

The sum of four consecutive even integers is 1284. The greatest of them is ?

**Solution :**

Let us assume smallest number as  $x$ .

As next consecutive even would be  $x + 2$ ,  $x + 4$ ,  $x + 6$

Sum of 4 consecutive digit is 1284

$$x + (x + 2) + (x + 4) + (x + 6) = 1284$$

$$4x + 12 = 1284$$

$$4x = 1272$$

$$x = 318$$

Smallest number is 318.

Largest number :  $x + 6 = 318 + 6 = 324$

**Answer is largest number is 324**

**Problem 16**

What is the sum of consecutive even numbers, the difference of whose square is 84?

**Solution :**

Let us assume smallest number as  $x$ .

Next even number would be  $x + 2$



Difference of square is 84

$$(x + 2)^2 - x^2 = 84$$

$$\text{use } (a + b)^2 = a^2 + 2ab + b^2$$

$$x^2 + 4x + 4 - x^2 = 84$$

$$4x + 4 = 84$$

$$4x = 80$$

$$x = 20$$

Smaller number =  $x = 20$

Next number =  $x + 2 = 20 + 2 = 22$

Numbers are 20 and 22

Sum of numbers =  $20 + 22 = 42$

**Answer is sum of number is 42**

**Problems on Ages**

Various competitive examinations ask questions regularly based on Problems on Ages

Many students face challenges in understanding and solving problems as they are not able to convert problem statements from words into equation of ages and ratio.

Let us take real life example and will try to understand concept of Problem of Ages.

**Real life example for Problems on Ages:**

*Let us assume Today your age is half of your father's age.*

- If your father age is 50 years then your age would be  $50/2 = 25$  years
- If your father's age is 60 years then your age would be  $60/2 = 30$  years
- If your father age is 100 years, your age would be  $100/2 = 50$  years

Ratio of Father's age to your age today is **2:1**

Problems would be based on Today's [Present] age, Past age or future age

If we are able to convert problem statement into equation we have solved 80% of the problem. We will practice initially for this before solving problem.

**Assume your Present age is  $x$  years** and we will understand meaning of different statements and its respective value.

| Statement              | Meaning in form of equation |
|------------------------|-----------------------------|
| Age after 10 years     | $x + 10$                    |
| Age after 5 years      | $x + 5$                     |
| Age 10 years ago       | $x - 10$                    |
| Age 25 years ago       | $x - 25$                    |
| 5 years Hence          | $x + 5$                     |
| 10 years Hence         | $x + 10$                    |
| Age 5 years back       | $x - 5$                     |
| Age 15 years back      | $x - 15$                    |
| 3 times of today's age | $3x$                        |

Assume:

Age of A : 20 Years

Age of B : 25 Years

As B's age is more than A,

B will be referred as **Elder** and A will be referred as **Younger**

Practice these similar statements and most of the time in numbers problem, you will either see 1 or more number of statement combination.

First step in problem would be convert problem statement into equation and then solve it.

We will solve problems now

**Problem 1:**

The ages of two persons differ by 16 years. If 6 years ago, elder one is 3 times as old as younger one, find their present ages

**Solution :**

Difference of present age for 2 Persons = 16 Years

6 Years ago, Elder age = 3 \* younger age

Present ages = ?

Let us assume age of Person1 as  $x$  years

As difference between Person1 and Person2 is 16 years

Person2 age = Person1 age - 16

Person2 age =  $x - 16$

So Person1 is elder and Person2 is Younger

6 Years ago

Person1 age =  $x - 6$

Person2 age =  $(x - 16) - 6 = x - 22$

It is given that 6 Years ago, Elder age = 3 \* younger age

$$x - 6 = 3(x - 22)$$

$$x - 6 = 3x - 66$$

$$2x = 60$$

$$x = 30$$

Person1 Present age =  $x = 30$  Years

Person2 Present age =  $x - 16 = 30 - 16 = 14$  Years.

*Note: You can cross-check your answer*

*Like present ages 30 and 14. So 6 years before, ages would be 24 and 8.*

*So elder was 3 times of younger. So our answer is correct.*

**Answer is Present ages of Persons are 30 Years and 14 Years**

**Problem 2:**

The product of ages of Ankit and Nikita is 240. If twice age of Nikita is more than Ankit's age by 4 years, what is Nikita's age?

**Solution :**

$$\text{Age of Ankit} \times \text{Age of Nikita} = 240$$

$$2 \times \text{age of Nikita} = \text{Age of Ankit} + 4$$

$$\text{Nikita Age} = ?$$

Let us assume age of Nikita is  $x$  years and age of Ankit is  $y$  years

$$\text{As it is given: } 2 \times \text{age of Nikita} = \text{Age of Ankit} + 4$$

$$2x = y + 4$$

$$y = 2x - 4$$

$$\text{Age of Ankit} \times \text{Age of Nikita} = 240$$

$$y \times x = 240$$

$$(2x - 4) \times x = 240$$

$$2x^2 - 4x - 240 = 0$$

$$x^2 - 2x - 120 = 0$$

$$x^2 - 12x + 10x - 120 = 0$$

$$x(x - 12) + 10(x - 12) = 0$$

$$(x + 10)(x - 12) = 0$$

$$\text{So } x + 10 = 0 \quad \text{or} \quad x - 12 = 0$$

$$x = -10 \text{ or } x = 12$$

we need to ignore -10 and consider  $x = 12$

Nikita present age = 12 years

**Answer is Nikita's age is 12 Years**

**Problem 3:**

Rohit was 4 times as old as his son 8 years ago. After 8 years, Rohit will be twice as old as his son. What are their present ages?

**Solution :**

$$8 \text{ Years ago: Rohit's age} = 4 \times \text{Son's age}$$

$$\text{After 8 years: Rohit's age} = 2 \times \text{Son's age}$$

Let us assume Present age of Rohit as  $x$  years and present age of his son  $y$  years

**8 Years ago:**

$$\text{Rohit's age} = x + 8 \text{ years}$$

$$\text{Son's age} = y + 8$$

Looking at given : 8 Years ago Rohit's age = 4 \* Son's age

$$x - 8 = 4 * (y - 8)$$

$$x - 8 = 4y - 32$$

$$x - 4y = -24 \text{ -----Equation1}$$

**After 8 Years:**

$$\text{Rohit's age} = x + 8 \text{ years}$$

$$\text{Son's age} = y + 8$$

Looking at given : After 8 year: Rohit's age = 2 \* Son's age

$$x + 8 = 2 * (y + 8)$$

$$x + 8 = 2y + 16$$

$$x - 2y = 8 \text{ -----Equation2}$$

$$x - 4y = -24 \text{ -----Equation1}$$

$$- \quad x - 2y = 8 \text{ -----Equation2}$$

$$- \quad + \quad -$$

---


$$-2y = -32$$

$$-2y = -32$$

$$y = 16 \quad [\text{Present age of Son}]$$

Put value of y in Equation1

$$x - 4y = -24$$

$$x - (4 * 16) = -24$$

$$x - 64 = -24$$

$$x = 40 \text{ [Present age of Rohit]}$$

**Answer is Present age of Rohit is 40 Years and Present age of son is 16 years**

**Problem 4:**

One year ago, the ratio of Gaurav's and Sachin's age was 6 : 7 respectively

Four years hence, this ratio would become 7 : 8. How old is Sachin?

**Solution :**

$$1 \text{ Year ago Gaurav Age : Sachin age} = 6 : 7$$

$$4 \text{ Years hence, Gaurav Age : Sachin age} = 7 : 8$$

$$\text{Sachin's Age} = ?$$

Let us assume Gaurav's present age as x years and present age of Sachin as y years

**1 Year ago:**

Gaurav's age = x – 1 years

Sachin's age = y – 1 years

As per Given 1 Year ago Gaurav Age · Sachin age = 6 · 7

$$\frac{x-1}{y-1} = \frac{6}{7}$$

$$7(x-1) = 6(y-1)$$

$$7x - 7 = 6y - 6$$

$$7x - 6y = 1 \text{ -----Equation1}$$

**Four years hence:**

Gaurav's age = x + 4 years

Sachin's age = y + 4 years

As per Given 4 Years hence, Gaurav Age · Sachin age = 7 · 8

$$\frac{x+4}{y+4} = \frac{7}{8}$$

$$8x + 32 = 7y + 28$$

$$8x - 7y = -4 \text{ -----Equation2}$$

We can not solve Equation1 and Equation2 directly as nothing is common to cancel

We will multiply Equation1 by 8 and Equation2 by 7

Equation1 · 7x – 6y = 1 [Multiply by 8 both sides]

$$56x - 48y = 8 \text{ -----Equation3}$$

Equation2 · 8x – 7y = -4 [Multiply by 7 both sides]

$$56x - 49y = -28 \text{ -----Equation4}$$

$$\begin{array}{r}
 56x - 48y = 8 \\
 - \quad 56x - 49y = -28 \\
 \hline
 \phantom{-} \quad \quad + \quad \quad +
 \end{array}$$

---


$$y = 36$$

**Answer is Sachin's present age is 36 years**

**Problem 5:**

At present, the ratio between ages of Arun and Deepak is 4 : 3. After 6 years, Arun's age will be 26 years. What is age of Deepak as present?

**Solution :**

Present age of Arun : Present age of Deepak = 4 : 3

After 6 years Arun age = 26 Years

Deepak's Present age = ?

After 6 years Arun's age will be 26 years.

So at Present Arun's age = 26 - 6 = 20 years

Given that Present age ratio Arun : Deepak = 4 : 3

$$\frac{\text{Present age of Arun}}{\text{Present age of Deepak}} = \frac{4}{3}$$

$$\frac{20}{\text{Present age of Deepak}} = \frac{4}{3}$$

Cross multiply to get answer

$$\text{Present age of Deepak} = \frac{20 \times 3}{4} = \frac{60}{4} = 15 \text{ Years}$$

**Answer is Deepak's Present age is 15 years.**



**Problem 6:**

Hitesh is 40 years old and Ronnie is 60 years old. How many years ago was their ratio 3:5?

- a) 5 years   b) 10 years   c) 20 years   d) 37 years

**Solution :**

In such kind of scenario it is always better to go with options rather than solving question.

How many years ago ratio was 3 : 5?

Hitesh present age = 40 years

Ronnie present age = 60 years

Option a: 5 years

5 years ago Hitesh age =  $40 - 5 = 35$

5 years ago Ronnie age =  $60 - 5 = 55$

$35 : 55 = 7 : 11$  [ So a is not answer]

Option b: 10 years

10 years ago Hitesh age =  $40 - 10 = 30$

10 years ago Ronnie age =  $60 - 10 = 50$

$30 : 50 = 3 : 5$  [ As it is matching with our expectation as 3: 5 ,so it is correct answer]

**Answer is 10 Years ago ratio of Hitesh and Ronnie's age was 3 : 5**

**Problem 7:**

A man is 24 years older than his son. In two years, his age will be twice age of his son. The present ages of Son is?

**Solution :**

Man present age = Son's present age + 24

After 2 years, Man's age = 2 \* son's age

Present age son = ?

Let us assume present age of son is x years

Man's Present age =  $x + 24$

After 2 years:

Son age =  $x + 2$

Man age =  $2(x + 2) = 2x + 4$  -----Equation 1

However Man's age after 2 years will be  $(x + 24) + 2 = x + 26$  -----

Equation2

Equation1 and Equation2 both are equal and nothing but man's age after 2 years

$$2x + 4 = x + 26$$

$$x = 22 \text{ [Present age of son]}$$

**Answer is Present age of Son is 22 years**

**Problem 8:**

The total age of A and B is 12 years more than total age of B and C. C is how many years younger than A?

**Solution :**

$$A \text{ age} + B \text{ age} = B \text{ age} + C \text{ age} + 12$$

$$C \text{ how many years younger} = ?$$

Assume A age : A years, B : B years and C : C years

It is given that:

$$A + B = B + C + 12$$

$$A = C + 12$$

This means C is 12 years younger than A.

**Answer is C is 12 years younger than A**

**Problem 9:**

The sum of ages of father and his son is 45 Years. Five years ago, the product of their ages was 34 years. The ages of son and father are respectively

**Solution :**

$$\text{Present Father age} + \text{Son age} = 45 \text{ years}$$

$$5 \text{ years ago product of ages} = 34$$

$$\text{Present age of Father's age and Son's age} = ?$$

Here we can take 2<sup>nd</sup> part to solve problem quickly

$$5 \text{ years ago product of ages} = 34$$

As to get product 34 there are only two options:

$$1) 17 \times 2$$

$$2) 34 \times 1$$

Let us take *option 1* .

5 years ago Father's age 17 and Son age 2

So present age Father's age =  $17 + 5 = 22$

Present age Son =  $2 + 5 = 7$

Total of 22 and 7 is not 45

*Option 2*

5 years ago Father's age 34 and Son age 1

So present age Father's age =  $34 + 5 = 39$

Present age Son =  $1 + 5 = 6$

Total of 39 and 6 is 45

**Answer is Present age of Father 39 Years and Son age is 6 years.**

**Work and Time**

Various competitive examinations ask questions regularly based on Work and Time. Work and Time is one of the highest rated topic by Paper setter for any competitive exam and it is lowest rated/most challenging topic from students point of view.

**Why Work and Time???**

Consider simple work of taking 20 printouts from your computer.

You may require 10 minutes to do this work. However, if same work to be done by your parents they may require 20 minutes to do it. If you ask same work to grandparents they may need 30 minutes to do same.

Consider simple work of writing letter manually on paper and post it to your friend. Here you can observe that your grandparents can do this work in 10 minutes [as they are used to with writing and posting letters]. However, as you are not very familiar or used to write letter manually and post it by visiting post office you may need 30 minutes to do same work.

Here you can observe that every-one is different in terms of efficiency and time required to do work. It also depends on area of expertise to that specific work for that person.

This is something really important in real time world while allocating people for work, doing estimates of any project etc. Therefore it is very important for us to solve Work and Time problems and apply them in our day to day work.

**Simple Method to solve Work and Time:**

We will be using simple LCM method to solve Work and Time problems.

This is different from conventional method which we generally study in our academics.

If we calculate LCM of numbers, we are almost 70% completion of problem.

Let us look at simple LCM examples first before looking at actual problems:

**Example 1:** Calculate LCM of 20, 30, 10

**Method 1: Divisional Method**

This is conventional method and we will keep dividing numbers till we get 1

|   |    |    |    |
|---|----|----|----|
| 2 | 20 | 30 | 10 |
| 5 | 10 | 15 | 5  |
|   | 2  | 3  | 1  |

$$\text{LCM} = 2 * 5 * 2 * 3 * 1 = 60$$

**Method 2: Orally by table method.**

Find greatest number from list for which LCM is to be identified.

Now look at table of this number and keep checking whether that number is divisible by other numbers in list.

Here largest number is 30. So we will write tables of 30 and table of 20, 10 as well.

We will write 30 table till we get the number which is completely divisible by 10 and 20

|           |           |           |
|-----------|-----------|-----------|
| 20        | 30        | 10        |
| 40        | <b>60</b> | 20        |
| <b>60</b> |           | 30        |
|           |           | 40        |
|           |           | 50        |
|           |           | <b>60</b> |

As you can see 30 is divisible by 10 but not by 20

However 60 is divisible by 10 and 20 both

**So 60 is LCM** [you can do this calculation orally in fraction of seconds if tables are learned 1 to 30]

**Example 2:** Calculate LCM of 25, 50

**Method 1: Divisional Method**

This is conventional method and we will keep dividing numbers till we get either 1 or all prime numbers

|   |    |    |
|---|----|----|
| 5 | 25 | 50 |
| 5 | 5  | 10 |
|   | 1  | 2  |

$$\text{LCM} = 5 * 5 * 2 * 1 * 2 = \mathbf{50}$$

**Method 2: Orally by table method**

Find greatest number from list for which LCM is to be identified.

Now look at table of this number and keep checking whether that number is divisible by other numbers in list.

Here largest number is 50. So we will write tables of 50 and table of 25 as well

We will write 50 table till we get the number which is completely divisible by 25.

|    |           |
|----|-----------|
| 25 | <b>50</b> |
| 50 |           |

As you can see 50 is divisible by 25

So **50 is LCM**

**Types of Problems:**

| Type   | Given  | To Find   |
|--------|--|---|
| Type 1 | A do in x days<br>B do in y days   | Together A & B will do in how much time         |
| Type 2 | A+B do in x days<br>A do in y days                                       | B alone will do in how much time                |
| Type 3 | A do in x days<br>B do in y days<br>If they work alternate days          | How much time will be required to complete work |
| Type 4 | A do in x days<br>A leaves job on 3 <sup>rd</sup> day and B worked alone | How much work pending / How much work is done   |

**Type1:****Problem 1:**

A does a work in 10 days and B does work in 15 days. In how many days they together can complete work?

**Solution :**

Let us assume Work -> Eating mangoes.

A takes 10 days to eat certain number of mangoes

B takes 15 days to eat certain number of mangoes

Calculate LCM of 10,15 -> LCM of 10 and 15 is 30.

| Person | Total Mango | Number of days | 1 day speed       |
|--------|-------------|----------------|-------------------|
| A      | 30          | 10             | $30/10 = 3$ Mango |
| B      | 30          | 15             | $30/15 = 2$ Mango |

So A eats 3 Mango in 1 day and B eats 2 Mango in 1 day

In order to do complete work [30 Mangos]

1 day Speed of A + B =  $3 + 2 = 5$

Time required to complete work =  $30/5 = 6$  Days

**Answer is 6 Days**

**Problem 2:**

A does a work in 10 hours. B does same work in 30 hours. C completes same work in 60 hours. How much time required to complete work if A, B and C worked together?

**Solution :**

Let us assume Work  $\rightarrow$  Eating mangoes.

A takes 10 hours to eat certain number of mangoes.

B takes 30 hours to eat certain number of mangoes.

C takes 60 hours to eat certain number of mangoes

Calculate LCM of 10, 30, 60  $\rightarrow$  LCM of 10, 30, 60 is 60

| Person | Total Mango | Number of hours | 1 Hour speed      |
|--------|-------------|-----------------|-------------------|
| A      | 60          | 10              | $60/10 = 6$ Mango |
| B      | 60          | 30              | $60/30 = 2$ Mango |
| C      | 60          | 60              | $60/60 = 1$ Mango |
| A+B+C  | 60          | $60/9$          | $6+2+1 = 9$ Mango |

A+B+C will require  $60/9$  hours to complete work

$$60/9 = 6\frac{6}{9}$$

**Answer is  $6\frac{6}{9}$  Hours**

**Type2:**

**Problem 1:**

The man can do a work in 5 days. With help of his son he can do that work in 3 days. How much time his son will take alone to complete work

**Solution :**

Calculate LCM of 5, 3  $\rightarrow$  LCM of 5, 3 is 15

| Person  | Total Mango | Number of days | 1 day speed      |
|---------|-------------|----------------|------------------|
| Man     | 15          | 5              | $15/5 = 3$ Mango |
| Man+Son | 15          | 3              | $15/3 = 5$ Mango |
| Son     | 15          | ?              | $5-3 = 2$ Mango  |



The speed of Man alone = 3 Mango in 1 day

The speed of Man +Son = 5 Mango in 1 day

Speed of Son alone = (Speed of Man +Son) – (Speed of Man alone)  
 $= 5 - 3 = 2$  Mango

Number of days son alone will require = Total mango / 1 Day speed  
 $= 15/2 = 7.5$  days

**Answer is 7.5 days**

**Problem 2:**

A & B together completes a work in 6 days B alone can do same work in 24 days How much A will take alone to complete work.

**Solution :**

Calculate LCM of 6, 24 -> LCM of 6, 24 is 24.

| Person | Total Mango | Number of days | 1 day speed       |
|--------|-------------|----------------|-------------------|
| A+B    | 24          | 6              | $24/6 = 4$ Mango  |
| B      | 24          | 24             | $24/24 = 1$ Mango |
| A      | 24          | ?              | $4-1 = 3$ Mango   |

The speed of A + B = 4 Mango in 1 day.

The speed of B alone = 1 Mango in 1 day

Speed of A alone = (Speed of A + B) – (Speed of B alone)  
 $= 4 - 1 = 3$  Mango

Number of days A alone will require = Total mango / 1 Day speed  
 $= 24/3 = 8$  days

**Answer is 8 days**

**Type3:**

**Problem 1:**

A completes work in 9 days and B completes same work in 12 days. If they work on alternate days, how much time they will require to finish work.

**Solution :**

Calculate LCM of 9, 12 -> LCM of 9, 12 is 36.

| Person | Total Mango | Number of days | 1 day speed       |
|--------|-------------|----------------|-------------------|
| A      | 36          | 9              | $36/9 = 4$ Mango  |
| B      | 36          | 12             | $36/12 = 3$ Mango |

A speed is 4 Mango per day

B speed is 3 Mango per day

As A and B are working on Alternate days

Total Mango to finish = 36

| Day             | Mango eaten | Pending Mango(36-eaten) |
|-----------------|-------------|-------------------------|
| 1 [A will work] | 4           | 32                      |
| 2 [B will work] | $4+3 = 7$   | 29                      |
| 3 [A will work] | $7+4 = 11$  | 25                      |
| 4 [B will work] | $11+3 = 14$ | 22                      |

Looking at this we can understand that in 2 days 7 Mango eaten

As we need to complete 36 Mango

36 is not completely divisible by 7 so we need to find number less than and nearest to 36 and should be completely divisible by 7.

To complete 35:

2 days - 7 Mango

?days - 35 Mango

Cross multiply to get answer

$$2 * 35 = 7 * ?$$

$$? = 70/7 = 10$$

35 Mango completed in 10 days and 1 still pending [as  $36 - 35 = 1$ ]

On 11<sup>th</sup> day 1 mango pending and A will be working

So A will require  $\frac{1}{4}$  time eat same [as in 1 day he eats 4].

Total days required =  $10 + \frac{1}{4}$

**Answer is  $10\frac{1}{4}$  Days**

**Problem 2:**

A completes work in 20 days and B completes same work in 30 days. If they work on alternate days, how much time they will require to finish work

**Solution :**

Calculate LCM of 20 30  $\rightarrow$  LCM of 20, 30 is 60

| Person | Total Mango | Number of days | 1 day speed       |
|--------|-------------|----------------|-------------------|
| A      | 60          | 20             | $60/20 = 3$ Mango |
| B      | 60          | 30             | $60/30 = 2$ Mango |

A speed is 3 Mango per day

B speed is 2 Mango per day

As A and B are working on Alternate days

Total Mango to finish 60.

| Day             | Mango eaten | Pending Mango(36-eaten) |
|-----------------|-------------|-------------------------|
| 1 [A will work] | 3           | 57                      |
| 2 [B will work] | $3+2 = 5$   | 55                      |

Looking at this we can understand that in 2 days 5 Mango eaten

As we need to complete 60 Mango

To complete 60:

2 days - 5 Mango

? days - 60 Mango

Cross multiply to get answer

$$2 * 60 = 5 * ?$$

$$? = 120/5 = 24 \text{ days will be required}$$

**Answer is 24 days**

**Type4:****Problem 1:**

A completes work in 15 days and B completes same work in 10 days They started working together but after 2 days work, B left the job and A continued alone to complete work In how much day pending work will be completed?

**Solution :**

Calculate LCM of 15, 10  $\rightarrow$  LCM of 15, 10 is 30

| Person | Total Mango | Number of days | 1 day speed       |
|--------|-------------|----------------|-------------------|
| A      | 30          | 15             | $30/15 = 2$ Mango |
| B      | 30          | 10             | $30/10 = 3$ Mango |

A speed is 2 Mango per day

B speed is 3 Mango per day

A+B speed for 1 day is 5 Mango

A +B worked together for 2 days

Number of mangoes eaten in this time =  $2 * 5 = 10$

Pending work [mango] = Total Mango – Completed work [mango]

Pending =  $30 - 10 = 20$

A as alone, To complete pending work : 20 mango

1 day - 2 Mango

? days - 20 Mango

Cross multiply to get answer

$1 * 20 = 2 * ?$

$? = 20/2 = 10$  days

To complete pending work by A alone 10 days will be required.

**Answer is 10 days**

**Problem 2:**

A completes work in 18 days and B completes same work in 15 days They started working together but after 5 days work, B left the job and A continued alone to complete work How much work pending and in how much days pending work will be completed?

**Solution :**

Calculate LCM of 18, 15  $\rightarrow$  LCM of 18, 15 is 90

| Person | Total Mango | Number of days | 1 day speed       |
|--------|-------------|----------------|-------------------|
| A      | 90          | 18             | $90/18 = 5$ Mango |
| B      | 90          | 15             | $90/15 = 6$ Mango |

A speed is 5 Mango per day

B speed is 6 Mango per day

A+B speed for 1 day is 11 Mango

A +B worked together for 5 days

Number of mangoes eaten in this time =  $11 * 5 = 55$

Pending work [mango] = Total Mango – Completed work [mango]

Pending =  $90 - 55 = 35$  Mango

Pending Work in terms of total work =  $35/90 = 7/18$

A as alone, To complete pending work : 35 mango

1 day - 5 Mango

? days - 35 Mango

Cross multiply to get answer

$1 * 35 = 5 * ?$

$? = 35/5 = 7$  days

**Answer is Pending work 7/18 and 7 days will be required to complete pending work**

### **Pipes and Cisterns**

Various competitive examinations ask questions regularly based on Pipes and Cisterns

If you have gone through my Work and Time document then solving Pipes and Cistern would definitely be very easy for you to solve

I would recommend you to go through work and time topic document once, before reading this. Work and Time, Pipes and Cisterns topics are almost similar to each other only the difference is that in Pipes and Cisterns, we need to look at negative work as well.

#### **What are Pipes and Cisterns???**

Let us look at 1 real life example to understand concept of Pipes and Cisterns.

Consider you have water storage tank of 1000 Liters on your terrace at 3<sup>rd</sup> floor

You utilize same water in your Kitchen, daily activities etc through different pipes

Assume you get water supply from Corporation at ground floor and you don't have facility of electric motor to transfer that water to Terrace tank.

So you fill water in bucket and go upstairs to fill tank. During this activity some water gets wasted due to leakage in bucket.

Someday magic happened and you got new Pipe in Tank itself which gets water from Corporation directly but your Tank also starts leaking at terrace itself

Now let us summarize things in terms of Pipes and Cisterns Problem

Terrace water Tank: **Tank / Cisterns**

Magic Pipe : **Inlet Pipe/Tap**

Water supply in Bath, Kitchen etc: **Outlet Pipe/Tap**

Bucket/Tank leakage: **Leakage in Tank**

**Simple Method to solve Pipes and Cisterns:**

We will be using simple LCM method to solve Work and Time problems

This is different from conventional method which we generally study in our academics

If we calculate LCM of numbers, we are almost 70% completion of problem.

Let us look at simple LCM examples first before looking at actual problems.

**Example 1:** Calculate LCM of 20, 30, 10

**Method 1: Divisional Method**

This is conventional method and we will keep dividing numbers till we get either 1 or all prime numbers

|  |   |    |    |    |
|--|---|----|----|----|
|  | 2 | 20 | 30 | 10 |
|  | 5 | 10 | 15 | 5  |
|  |   | 2  | 3  | 1  |

$$\text{LCM} = 2 * 5 * 2 * 3 * 1 = 60$$

**Method 2: Orally by table method.**

Find greatest number from list for which LCM is to be identified.

Now look at table of this number and keep checking whether that number is divisible by other numbers in list.

Here largest number is 30. So we will write tables of 30 and table of 20, 10 as well.

We will write 30 table till we get the number which is completely divisible by 10 and 20

|           |           |           |
|-----------|-----------|-----------|
| 20        | 30        | 10        |
| 40        | <b>60</b> | 20        |
| <b>60</b> |           | 30        |
|           |           | 40        |
|           |           | 50        |
|           |           | <b>60</b> |

As you can see 30 is divisible by 10 but not by 20

However 60 is divisible by 10 and 20 both

**So 60 is LCM** [you can do this calculation orally in fraction of seconds if tables are learned 1 to 30]

**Example 2:** Calculate LCM of 25, 50

**Method 1: Divisional Method**

This is conventional method and we will keep dividing numbers till we get either 1 or all prime numbers

|   |    |    |
|---|----|----|
| 5 | 25 | 50 |
| 5 | 5  | 10 |
|   | 1  | 2  |

$$\text{LCM} = 5 * 5 * 2 * 1 * 2 = 50$$

**Method 2** Orally by table method

Find greatest number from list for which LCM is to be identified.

Now look at table of this number and keep checking whether that number is divisible by other numbers in list.

Here largest number is 50. So we will write tables of 50 and table of 25 as well

We will write 50 table till we get the number which is completely divisible by 25.

|           |           |
|-----------|-----------|
| 25        | <b>50</b> |
| <b>50</b> |           |

As you can see 50 is divisible by 25

**So 50 is LCM**



**Types of Problems:**

| Type   | Given  | To Find  |
|--------|--|--|
| Type 1 | Pipe A fills tank in x hours<br>Pipe B fills tanks n y hours   | Together A & B will fill tank in how much time                     |
| Type 2 | Pipe A fills tank in x hours<br>Pipe B empties tank n y hours  | Together A & B will fill tank in how much time tank will be filled |
| Type 3 | Pipe A fills tank in x hours<br>Due to leak it takes more time | How much time will be required to empty tank completely            |
| Type 4 | Miscellaneous  |  |

**Type1:****Problem 1:**

Pipe A fills tank in 10 hours and Pipe B fills tank in 15 hours. In how much time tank will fill completely if both pipes are opened together?

**Solution :**

Pipe A takes 10 hours to fill tank

Pipe B takes 15 hours to fill tank

First we will try to calculate what is capacity of tank in liters' which is nothing but LCM

Calculate LCM of 10,15 -> LCM of 10 and 15 is 30.

| Pipe | Tank Capacity | Time to fill tank(hours) | Speed for 1hour    |
|------|---------------|--------------------------|--------------------|
| A    | 30            | 10                       | $30/10 = 3$ liters |
| B    | 30            | 15                       | $30/15 = 2$ liters |

So A fills 3 liters in 1 hour and 2 liters in 1 hour

In order to fill complete tank [30 Liters]

1 hr Speed for Pipe A + B =  $3 + 2 = 5$

Time required to fill complete tank =  $30 / 5 = 6$  Hours

**Answer is 6 Hours**

**Problem 2:**

Pipe A fills tank in 10 hours Pipe B fills tank in 30 hours Pipe C fills tank in 60 hours. How much time required to fill tank completely if A, B and C worked together?

**Solution :**

Pipe A takes 10 hours to fill tank

Pipe B takes 30 hours to fill tank

Pipe C takes 60 hours to fill tank

Calculate LCM of 10, 30, 60  $\rightarrow$  LCM of 10, 30, 60 is 60.

Total tank capacity = 60 Liters

| Pipe  | Tank Capacity | Time to fill tank(hours) | 1 Hour speed |
|-------|---------------|--------------------------|--------------|
| A     | 60            | 10                       | $60/10 = 6$  |
| B     | 60            | 30                       | $60/30 = 2$  |
| C     | 60            | 60                       | $60/60 = 1$  |
| A+B+C | 60            | $60/9$                   | $6+2+1 = 9$  |

A+B+C will require  $60/9$  hours to fill tank completely

$$60/9 = 6\frac{6}{9}$$

**Answer is  $6\frac{6}{9}$  Hours**

**Problem 3:**

Pipe A fills tank in 20 minutes and Pipe B fills tank in 30 minutes. How much time tank will fill completely if both pipes are opened together?

**Solution :**

Pipe A takes 20 hours to fill tank.

Pipe B takes 30 hours to fill tank

Calculate LCM of 20, 30 → LCM of 20 and 30 is 60

Tank capacity :

| Pipe  | Tank Capacity | Time to fill tank(minutes) | Speed for 1minute |
|-------|---------------|----------------------------|-------------------|
| A     | 60            | 20                         | $60/20 = 3$       |
| B     | 60            | 30                         | $60/30 = 2$       |
| A + B | 60            | $60/5 = 12$                | $3 + 2 = 5$       |

**Answer is 12 Minutes**

### Type2:

#### **Problem 1:**

A Cistern can be filled by tap in 4 hours while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously then after how much time will cistern get filled?

**Solution :**

Pipe A takes 4 hours to fill tank.

Pipe B takes 9 hours to fill tank

**We need to use minus sign in speed of B as it is outlet pipe.**

Calculate LCM of 4, 9 → LCM of 4, 9 is 36

So Cistern capacity is 36 Liters.

| Pipe | Cistern Capacity | Time to fill/empty(hours) | 1 hour speed   |
|------|------------------|---------------------------|----------------|
| A    | 36               | 4                         | $36/4 = 9$     |
| B    | 36               | 9                         | $36/9 = -4$    |
| A+B  | 36               | ?                         | $9 + (-4) = 5$ |

As you can see 1<sup>st</sup> pipe is filling tank and 2<sup>nd</sup> pipe is outlet pipe so we are using minus sign for 2<sup>nd</sup> pipe

In 1 hr total speed is 5

To fill complete tank that is 36 Litres

$$\frac{36}{5} = 7\frac{1}{5}$$

$1/5$  means  $60/5 = 12$  Minutes

**Answer is 7.2 Hours or 7 Hours 12 minutes**

**Problem 2:**

Two pipes can fill tank in 10 hours and 12 hours respectively, while 3<sup>rd</sup> pipe empties the full tank in 20 hours. If all three pipes are operated simultaneously then in how much time tank will be filled completely?

**Solution :**

Pipe A takes 10 hours to fill tank

Pipe B takes 12 hours to fill tank

Pipe C takes 20 hours to fill tank

**We need to use minus sign in speed of C as it is outlet pipe.**

Calculate LCM of 10, 12, 20  $\rightarrow$  LCM of 10, 12, 20 is 60

So Tank capacity is 60 Liters

| Pipe  | Cistern Capacity | Time to fill/empty(in hours) | 1 hour speed       |
|-------|------------------|------------------------------|--------------------|
| A     | 60               | 10                           | $60/10 = 6$        |
| B     | 60               | 12                           | $60/12 = 5$        |
| C     | 60               | 20                           | $60/20 = -3$       |
| A+B+C | 60               | ?                            | $6 + 5 + (-3) = 8$ |

As C pipe empties pipes we have used minus sign

In 1 hr if all opened together 8 liters water would be filled.

In order to fill complete tank that is 60Liters

$$\frac{60}{8} = \frac{15}{2} = 7\frac{1}{2}$$

$\frac{1}{2}$  means  $60/2 = 30$  minutes

**Answer is 7.5 Hours or 7 Hours 30 minutes**

**Type 3:**

**Problem 1:**

An electric pump can fill tank in 3 hours. Because of leak in the tank, it took  $3\frac{1}{2}$  hours to fill tank. If tank is full, how much time it will leak take to empty it.

**Solution:**

Pipe A takes 3 hours to fill tank.

Pipe A with leakage (Assume leakage as B) takes  $3\frac{1}{2}$  hours =  $\frac{7}{2}$  hours

Here we need to find how much time leakage will require to empty completely filled pipe

Let us calculate LCM of 3 and  $\frac{7}{2}$

As it sometimes become challenging to find LCM of fraction, let us see simple method

Multiply both number for which LCM to be find.

After that multiplication contains fraction multiply same with denominator to get natural number

$$3 * \frac{7}{2} = \frac{21}{2}$$

Here we have denominator and we cant take it as LCM

So

$$\frac{21}{2} * 2 = 21$$

LCM of 3 and  $\frac{7}{2}$  is 21

Now let us use same method what we used earlier

Please note that B is leakage pipe so it will have minus sign

| Pipe | Cistern Capacity | Time to fill/empty(in hours) | 1 hour speed   |
|------|------------------|------------------------------|----------------|
| A    | 21               | 3                            | $21/3 = 7$     |
| A+B  | 21               | $7/2$                        | $21/(7/2) = 6$ |
| B    | 21               | 21                           | $7 - 6 = 1$    |
|      |                  |                              |                |

Speed of A = 7

Speed of A with Leakage B = 6

So speed of Leakage B is 1

In order to empty filled pipe leakage will need  $21 / 1 = 21$  Hours

**Answer is 21 Hours**

### Problem 2:

A pump can fill tank with water in 2 hours. Because of leak, it took  $2\frac{1}{3}$  hours to fill the tank. The leak can drain all water in tank in how much time?

### Solution:

Pipe A takes 2 hours to fill tank.

Pipe A with leakage (Assume leakage as 2) takes  $2\frac{1}{3}$  hours =  $\frac{7}{3}$  hours

Here we need to find how much time leakage will require to empty completely filled pipe

Let us calculate LCM of 2 and  $\frac{7}{3}$

As it sometimes become challenging to find LCM of fraction, let us see simple method

Multiply both number for which LCM to be find.

After that multiplication contains fraction multiply same with denominator to get natural number

$$2 * \frac{7}{3} = \frac{14}{3}$$

Here we have denominator and we cant take it as LCM

So

$$\frac{14}{3} \times 3 = 14$$

LCM of 2 and  $\frac{7}{3}$  is 14

Now let us use same method what we used earlier

Please note that B is leakage pipe so it will have minus sign

| Pipe | Cistern Capacity | Time to fill/empty | 1 hr speed             |
|------|------------------|--------------------|------------------------|
| A    | 14               | 2                  | $14/2 = 7$             |
| A+B  | 14               | $7/3$              | $14/(\frac{7}{3}) = 6$ |
| B    | 14               | 14                 | $7 - 6 = -1$           |

Speed of A = 7

Speed of A with Leakage B = 6

So speed of Leakage B is 1

In order to empty filled pipe leakage will need  $14 / 1 = 14$  Hours

**Answer is 14 Hours**

**Type 4:**

**Problem 1:**

Two pipes A and B can fill a tank in 6 hours and 4 hours respectively. If they are opened on alternate hours and if Pipe A is opened first, in how many hours tank will be full?

**Solution:**

Pipe A takes 6 hours to fill tank.

Pipe B takes 4 hours to fill tank

Calculate LCM of 6, 4  $\rightarrow$  LCM of 6 and 4 is 12.

Tank capacity =

| Pipe | Tank Capacity | Time to fill tank(hours) | Speed for 1hour |
|------|---------------|--------------------------|-----------------|
| A    | 12            | 6                        | $12/6 = 2$      |
| B    | 12            | 4                        | $12/4 = 3$      |

We have calculated 1 hour speed of pipe A and B. However we can't use it directly to solve problem as pipes are opened alternatively starting from A.

| Hour                    | Tank filled (in liters) | Tank remaining (in liters) |
|-------------------------|-------------------------|----------------------------|
| 1 [Pipe A will be open] | 2                       | 10                         |
| 2 [Pipe B will be open] | $2+3 = 5$               | 7                          |
| 3 [Pipe A will be open] | $5+2 = 7$               | 5                          |
| 4 [Pipe B will be open] | $7+3 = 10$              | 2                          |

In 4 hours, 10 Liters will be filled and 2 Liters is remaining.

And now Pipe A will open and Pipe A speed is 2 Liters per hour.

In order to fill 2 liters by pipe A:

$$\frac{2}{2} = 1$$

$$4 + 1 = 5 \text{ Hours}$$

**Answer is 5 Hours**

### Problem 2:

Two pipes A and B can fill tank in 15 hours and 20 hours respectively while third pipe C can empty full tank in 25 hours. All 3 pipes are opened in beginning. After 10 hours, C is closed. In how much time will tank be full?

**Solution :**

Pipe A takes 15 hours to fill tank

Pipe B takes 20 hours to fill tank

Pipe C takes 25 hours to empty full tank

**We need to use minus sign in speed of C as it is outlet pipe.**

Calculate LCM of 15, 20, 25  $\rightarrow$  LCM of 15, 20, 25 is 300



Tank capacity is 300 Liters

| Pipe  | Tank Capacity | Time to fill/empty(in hours) | 1 hour speed           |
|-------|---------------|------------------------------|------------------------|
| A     | 300           | 15                           | $300/15 = 20$          |
| B     | 300           | 20                           | $300/20 = 15$          |
| C     | 300           | 25                           | $300/25 = -12$         |
| A+B+C |               |                              | $20 + 15 + (-12) = 23$ |
| A+B   |               |                              | $20 + 15 = 35$         |

All 3 pipes initially opened

Speed during this time **23**

For 10 hours all pipes opened:

In 1 hour speed of all 3 pipes together is 23

So in 10 hours:  $23 * 10 = 230$  Liters filled

Post 10 hours pending tank

$300 - 230 = 70$  Liters

Now C pipe is closed so only A and B is opened

Speed of A+B is 35

In order to complete pending 70 Liters:

$$\frac{70}{35} = 2$$

Total time =  $10 + 2 = 12$  Hours

**Answer is 12 Hours**

### Problem 3:

Two pipes A and B can fill a tank in 24 min and 32 min respectively. If both pipes are opened simultaneously after how much time B should be closed so that tank full in 18 minutes

**Solution:**

Pipe A takes 24 minutes to fill tank

Pipe B takes 32 minutes to fill tank

We need to complete tank in 18 minutes by closing B after some interval

Calculate LCM of 24, 32,  $\rightarrow$  LCM of 24, 32 is 96

Tank capacity is 96 Liters

| Pipe | Tank Capacity | Time to fill/empty( minutes) | 1 minute speed |
|------|---------------|------------------------------|----------------|
| A    | 96            | 24                           | $96/24 = 4$    |
| B    | 96            | 32                           | $96/32 = 3$    |
| A+B  |               |                              | $4 + 3 = 7$    |

A+B speed is 7

Let us assume A and B both were opened for 18 minutes

In 18 minutes:

$18 * 7 = 126$  Liters of water would be filled

However our tank is of 96 Liters only

In order to find when B should be closed we need to see what extra water filled by Pipe B

Extra water by B  $= 126 - 96 = 30$  Liters

As B speed is 3 it will take  $(30/3 = 10)$  10 minutes to do this extra water so we need to close B 10 minutes before 18 minutes

So  $18 - 10 = 8$  minutes

**Answer is we should close B after 8 minutes so tank will be filled in 18 minutes.**

## Partnership

Various competitive examinations ask questions regularly based on Partnership.

Many students face challenges in understanding and solving problems as they face challenge while working in Ratio and proportion

Let us take real life example and will try to understand first what is partnership and how we need to find ratio and proportion. Once you understand concept clearly, you can solve Partnership problems easily

### What is Partnership?

Partnership is something where more than 1 person effort is needed to complete goal.

Imagine world cup match India v/s Australia and you have target of 300 Runs. Without doing multiple good partnership, can we win the game? Obviously not!!!

Let us use same example to understand Partnership concept.

We will look at scorecards for multiple cricket matches where Sachin and Sehwag did opening partnership for India and we will award them some amount as rewards for runs they did during partnership.

| Match | Partnership (runs) | Sachin's contribution | Sehwag's contribution | Reward per run(Rs) | Sachin's reward     | Sehwag's Reward     |
|-------|--------------------|-----------------------|-----------------------|--------------------|---------------------|---------------------|
| 1     | 100                | 65                    | 35                    | 1                  | $65 * 1 = 65$       | $35 * 1 = 35$       |
| 2     | 200                | 110                   | 90                    | 1                  | $110 * 1 = 110$     | $90 * 1 = 90$       |
| 3     | 160                | 100                   | 60                    | 10                 | $100 * 10 = 1000$   | $60 * 10 = 600$     |
| 4     | 300                | 160                   | 140                   | 20                 | $160 * 20 = 3200$   | $140 * 20 = 2800$   |
| 5     | 90                 | 50                    | 40                    | 100                | $50 * 100 = 5000$   | $40 * 100 = 4000$   |
| 6     | 80                 | 60                    | 20                    | 1000               | $60 * 1000 = 60000$ | $20 * 1000 = 20000$ |
| 7     | 50                 | 25                    | 25                    | 500                | $25 * 500 = 12500$  | $25 * 500 = 12500$  |

Looking at table:

- 1) Individual person gets reward/ profit based on individual contribution
- 2) Total reward is divided based on efforts/contribution done by individual person

Now we try to find out ratio of efforts and subsequently ratio of rewards.

What is ratio?

*Let us assume Today your age is half of your father's age.*

- If your father age is 50 years then your age would be  $50/2 = 25$  years
- If your father's age is 60 years then your age would be  $60/2 = 30$  years
- If your father age is 100 years, your age would be  $100/2 = 50$  years

Let us represent same in ratio:

$$\frac{\text{Father's age}}{\text{You 's age}} = \frac{50}{25} = \frac{60}{30} = \frac{100}{50} = \frac{2}{1} \text{ it is nothing but } 2:1$$

Ratio of Father's age to your's age today is **2:1**

We will refer same table of Partnership of Sachin and Sehwag and we will calculate ratio of their contribution and accordingly ratio of their rewards

| Match | Sachin's contribution | Sehwag's contribution | Ratio of Contribution Sachin : Sehwag | Ratio of Rewards Sachin : Sehwag |
|-------|-----------------------|-----------------------|---------------------------------------|----------------------------------|
| 1     | 65                    | 35                    | $65/35 = 13 : 7$                      | $65/35 = 13 : 7$                 |
| 2     | 110                   | 90                    | $110/90 = 11 : 9$                     | $110/90 = 11 : 9$                |
| 3     | 100                   | 60                    | $100/60 = 5 : 3$                      | $1000/600 = 5 : 3$               |
| 4     | 160                   | 140                   | $160/140 = 8 : 7$                     | $3200/2800 = 8 : 7$              |
| 5     | 50                    | 40                    | $50/40 = 5 : 4$                       | $5000/4000 = 5 : 4$              |
| 6     | 60                    | 20                    | $60/20 = 3 : 1$                       | $60000/2000 = 3 : 1$             |
| 7     | 25                    | 25                    | $25/25 = 1 : 1$                       | $12500/12500 = 1 : 1$            |

*As you can observe in table, ratio of contribution is always same as ratio of Rewards.*

Now we will co-relate terms to Partnership problems

**Summary:**

| Real life example(Sachin -Sehwag partnership) | Meaning in Partnership problems |
|---|---------------------------------|
| Contribution in Partnership                   | Investment                      |
| Ratio of contribution(Runs)                   | Ratio of Investment             |
| Reward  | Profit                          |
| Individual reward                             | Individual share of Profit.     |

**Types of Problems:**

| Type   | Given  | To Find                                |
|--------|--|--|
| Type 1 | Investment amount for 2 or more persons (A, B, C)  | Ratio of Profit                        |
| Type 2 | Investment amount 2 or more persons (A, B, C) for same time period and Total Profit      | Individual Profit share                |
| Type 3 | Investment amount 2 or more persons (A, B, C) for different time period and Total Profit | Individual Profit share                |
| Type 4 | Investment amount 2 or more persons (A, B, C) and Individual Profit for 1 person         | Total Profit / Individual Profit share |
| Type 5 | Miscellaneous  |  |

**Type1:****Problem 1:**

P and Q started a business investing 85000 Rs and 15000 Rs respectively  
In what ratio the profit earned after 2 years be divided between P and Q respectively?

**Solution :**

Investment of P = 85000 Rs

Investment of Q = 15000 Rs

Ratio of Profit of P : Q = ?

Here, individual investment of P and Q is given.

$$\begin{aligned}\text{Ratio of Profit for P : Q} &= \frac{\text{Investment of P}}{\text{Investment of Q}} \\ &= \frac{85000}{15000} = \frac{85}{15} = \frac{17}{3}\end{aligned}$$

**Answer is Ratio of Profit for P :Q is 17 : 3**

**Problem 2:**

A, B and C started a business by investing 120000 Rs, 135000 Rs and 150000 Rs respectively Find out ratio of Profit for A, B, C respectively.

**Solution :**

Investment of A = 120000 Rs

Investment of B = 135000 Rs

Investment of C = 150000 Rs

Ratio of Profit of A : B : C = ?

Here, individual investment of A, B and C is given

Ratio of Profit for A : B : C = Investment of A : Investment of B : Investment of C

$$= 120000 : 135000 : 150000$$

We need to simplify them so that we can find answer

$$120000 : 135000 : 150000$$

$$120 : 135 : 150 \quad [\text{cancelled all 3 zeroes}]$$

$$8 : 9 : 10 \quad [\text{if you know table of 15 you can see all of them divisible by 15}]$$

**Answer is Ratio of Profit for A :B : C is 8 : 9 : 10**

**Type2:****Problem 1:**

Sanjay and Deepak started a business investing 22500 Rs and 35000 Rs respectively. Out of total profit of 13800 Rs, Deepak share is?

**Solution :**

Investment of Sanjay = 22500 Rs

Investment of Deepak = 35000 Rs

Total profit : 13800 Rs

Deepak share : ?

Here, individual investment of Sanjay and Deepak are given

Ratio of Profit for Sanjay : Deepak =  $\frac{\text{Investment of Sanjay}}{\text{Investment of Deepak}}$

$$= \frac{22500}{35000} = \frac{225}{350} = \frac{9}{14}$$

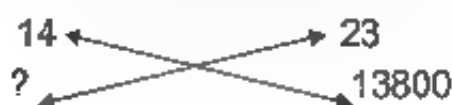
Ratio of Profit for Sanjay : Deepak is 9 : 14

This means that if Sanjay gets 9 Rs profit then Deepak will get 14 Rs

So total profit = 9 + 14 = 23 Rs

It is given that profit is 13800. So we need to cross multiply to get answer we need to find share of Deepak:

Deepak share                      Total share



$$\text{Deepak share} = \frac{14 \times 13800}{23} = \frac{14 \times 600}{1} = 8400$$

**Answer is Deepak's share of Profit is 8400 Rs**

**Problem 2:**

A, B, C enter into a partnership investing 35000 Rs, 45000 Rs and 55000 Rs respectively. The respective shares of A, B and C in annual profit of 40500 Rs are?

**Solution :**

Investment of A = 35000 Rs

Investment of B = 45000 Rs

Investment of C = 55000 Rs

Annual profit = 40500 Rs

Individual Profit of A , B and C = ?

Ratio of Profit for A : B : C = Investment of A : Investment of B : Investment of C

$$= 35000 : 45000 : 55000$$

$$= 35 : 45 : 55$$

$$= 7 : 9 : 11$$

This means if A gets Profit 7 Rs then B will get 9 Rs, C will get 11 Rs

So total  $7 + 9 + 11 = 27$  Rs

To identify common factor of multiplication [ assume x ]:

$$27x = 40500$$

$$x = \frac{40500}{27} = \frac{13500}{9} = 1500 \text{ [ common multiplication factor to get actual profit ]}$$

$$A : B : C = 7 : 9 : 11$$

.

A's Profit =  $7 * 1500 = 10500$  Rs

B's Profit =  $9 * 1500 = 13500$  Rs

C's Profit =  $11 * 1500 = 16500$  Rs

**Answer is A, B, C profits are 10500 Rs, 13500 Rs and 16500 Rs**

**Problem 3:**

A , B, C enter into a partnership investing 120000 Rs 135000 Rs and 150000 Rs respectively Find the share of each out of annual profit of Rs 56700

**Solution :**

Investment of A = 120000 Rs

Investment of B = 135000 Rs

Investment of C = 150000 Rs

Annual profit = 56700 Rs

Individual Profit of A , B and C = ?



Ratio of Profit for A : B : C = Investment of A : Investment of B : Investment of C

$$= 120000 : 135000 : 150000$$

$$= 120 : 135 : 150$$

$$= 8 : 9 : 10$$

This means if A gets Profit 8 Rs then B will get 9 Rs, C will get 10 Rs

So total  $8 + 9 + 10 = 27$  Rs

To identify common factor of multiplication [ assume x ]

$$27x = 56700$$

$$x = \frac{56700}{27} = \frac{6300}{3} = 2100 \text{ [ common multiplication factor to get actual profit ]}$$

$$A : B : C = 8 : 9 : 10$$

.

$$A's \text{ Profit} = 8 * 2100 = 16800 \text{ Rs}$$

$$B's \text{ Profit} = 9 * 2100 = 18900 \text{ Rs}$$

$$C's \text{ Profit} = 10 * 2100 = 21000 \text{ Rs}$$

**Answer is A, B, C profits are 16800 Rs, 18900 Rs and 21000 Rs**

### Type 3

#### Problem 1:

A, B, C starts a business each investing 20000 Rs, After 5 months A withdrew 5000 Rs, B withdrew 4000 Rs and C invests 6000 Rs more. At end of year, total profit of 69900 Rs was recorded. Find the share of each.

**Solution :**

Investment of A = 20000 Rs [ post 5 month withdraw 5000 ]

Investment of B = 20000 Rs [ post 5 month withdraw 4000 ]

Investment of C = 20000 Rs [ post 5 month invests 6000 more ]

Annual profit = 69900 Rs

Individual Profit of A, B and C = ?

As you can see here contribution from start of business to end of year [Profit time] we need to calculate investment yearly based on intermed ate withdraw / more investment]

As time is 1 year [12 months]

A withdraw 5000 Rs after 5 months , so for last 7 month investment to be reduced

$$\begin{aligned} \text{A's investment in year} &= (20000 * 12) - (5000 * 7) \\ &= 240000 - 35000 = \mathbf{205000 \text{ Rs}} \end{aligned}$$

$$\begin{aligned} \text{B's investment in year} &= (20000 * 12) - (4000 * 7) \\ &= 240000 - 28000 = \mathbf{212000 \text{ Rs}} \end{aligned}$$

$$\begin{aligned} \text{C's investment in year} &= (20000 * 12) + (6000 * 7) \\ &= 240000 + 42000 = \mathbf{282000 \text{ Rs}} \end{aligned}$$

As you can see for withdraw we will use minus sign (-) and for more investment (+)

Ratio of Profit for A : B : C = Investment of A : Investment of B : Investment of C

$$\begin{aligned} &= 205000 : 212000 : 282000 \\ &= 205 : 212 : 282 \end{aligned}$$

If A gets Profit 205 Rs then B will get 212 Rs and C will get 282 Rs

$$\text{Total profit} = 205 + 212 + 282 = 699$$

To identify common factor of multiplication [ assume x]:

$$699x = 69900$$

$$x = 100 \text{ [ Common factor is 100]}$$

$$\text{Ratio of Profit for A : B : C} = 205 : 212 : 282$$

$$\text{A's profit} = 205 * 100 = 20500 \text{ Rs}$$

$$\text{B's profit} = 212 * 100 = 21200 \text{ Rs}$$

$$\text{C's profit} = 282 * 100 = 28200 \text{ Rs}$$

**Answer is A, B, C profits are 20500 Rs, 21200 Rs and 28200 Rs**

**Problem 2**

Reema and Sayali are partners in business. Reema invests 35000 Rs for 8 months and Sayali invests 42000 Rs for 10 months. Out of profit 31570 Rs Reema's share is ?

**Solution :**

Investment of Reema = 35000 Rs [For 8 months]

Investment of Sayali = 42000 Rs [For 10 months]

Profit = 31570 Rs

Individual Profit of Reema = ?

As you can see here contribution is for different time period, we need to find actual investment done by Reema and Sayali.

Investment of Reema = 35000 for 8 months

$$= 35000 \times 8 = \mathbf{280000 \text{ Rs}}$$

Investment of Sayali = 42000 for 10 months

$$= 42000 \times 10 = \mathbf{420000 \text{ Rs}}$$

Ratio of Profit for Reema : Sayali = Investment of Reema : Investment of Sayali

$$= 280000 : 420000$$

$$= 28 : 42$$

$$= 2 : 3$$

If Reema gets 2 Rs Profit then Sayali will get 3 Rs

So total is 5 Rs Profit.

Reema share

Total share



$$\text{Reema share} = \frac{2 \times 31570}{5} = \frac{2 \times 6314}{1} = 12628 \text{ Rs}$$

**Answer is Reema's share of Profit is 12628 Rs**

## Type 4

## Problem 1

Four milkmen rented a pasture. A grazed 24 cows for 3 months, B 10 cows for 5 months, C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is 720 Rs, find total rent?

## Solution

A's usage of place = 24 cows for 3 months

B's usage of place = 10 cows for 5 months

C's usage of place = 35 cows for 4 months

D's usage of place = 21 cows for 3 months

A's share of rent : 720 Rs

Total rent ?

We will calculate individual contribution of investment first:

A's usage = 24 cows for 3 month

$$= 24 \times 3 = 72$$

B's usage = 10 cows for 5 month

$$= 10 \times 5 = 50$$

C's usage = 35 cows for 4 month

$$= 35 \times 4 = 140$$

D's usage = 21 cows for 3 month

$$= 21 \times 3 = 63$$

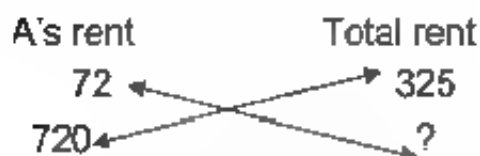
They will pay rent as per area they will be using

Ratio of Rent = Ratio of area of usage A : B : C : D

$$= 72 : 50 : 140 : 63$$

When A will pay 72 Rs rent, then B will pay 50 Rs, C will pay 140 Rs and D will pay 63 Rs

So total  $72 + 50 + 140 + 63 = 325$  Rs



$$\text{Total rent} = \frac{720 \times 325}{72} = \frac{10 \times 325}{1} = 3250$$

**Answer is Total rent is 3250 Rs**

### Problem 2

A, B and C started shop by investing 27000 Rs, 72000 Rs and 81000 Rs respectively. At end of year profit was distributed among them. If C's profit is 36000 Rs, then what is total profit?

### Solution

A's investment : 27000 Rs

B's investment : 72000 Rs

C's investment : 81000 Rs

C's share of profit : 36000 Rs

Total profit : ?

Ratio of Profit for A : B : C = Investment of A : Investment of B : Investment of C

$$= 27000 : 72000 : 81000$$

$$= 27 : 72 : 81$$

$$= 3 : 8 : 9$$

This means if A gets Profit 3 Rs then B will get 8 Rs, C will get 9 Rs

So total  $3 + 8 + 9 = 20$  Rs

C's Profit                      Total profit



$$\text{Total Profit} = \frac{20 \times 36000}{9} = \frac{20 \times 400}{1} = 8000$$

**Answer is Total profit is 8000 Rs**

**Type 5****Problem 1**

X and Y invested in a bus ness. They earned some profit which they divided n ratio of 2: 3. If X invested 4000 Rs, the amount invested by Y is?

**Solution**

Profit ratio of X: Y = 2: 3

X's Investment : 40000 Rs

Y's Investment: ?

We know that profit ratio is same as Investment ratio. So here profit ratio is given and we need to find out Original investment

$$\frac{\text{X Profit}}{\text{Y Profit}} = \frac{\text{X Investment}}{\text{Y Investment}}$$

$$\frac{2}{3} = \frac{40000}{\text{Y Investment}}$$

Cross multiplying:

$$\text{Y's investment} = \frac{40000 * 3}{2} = 20000 * 3 = 60000$$

**Answer is Y's investment is 60000 Rs**

**Problem 2**

A invested 76000 Rs in bus ness. After few months B joined him with 57000 Rs. At the end of year, total profit was divided between them in the ratio of 2 : 1. After how many months did B join?

**Solution**

A's investment throughout year = 76000 Rs

B's investment after some time = 57000 Rs

Profit ratio A : B = 2 : 1

After how much time B should have joined = ?

We need to first calculate exact investment for year for both A and B

For A it is simple as it was for throughout year (12 months)

A's investment = 76000 \* 12 = 912000 Rs

Let us assume B enters after x months, so total investment for B will be  
(12 - x) \* 57000

$$\frac{\text{A Profit}}{\text{B Profit}} = \frac{\text{A Investment}}{\text{B Investment}}$$

$$\frac{2}{1} = \frac{91200}{(12-x) * 57000}$$

$$2 * 57000 * (12 - x) = 912000$$

$$12 - x = \frac{912000}{2 * 57000} = \frac{912}{2 * 57} = \frac{48}{2 * 3} = 8$$

$$12 - x = 8$$

$$x = 4$$

**Answer is B joined after 4 months**

## Time and Distance

Various competitive examinations ask questions regularly based on Time and Distance

Many students face challenges in understanding and solving problems. This is a thing which we really use a lot while driving bike, car or travelling from one city to other etc

Most of people would say remember formula and then you can solve problems. However, we are able to remember limited amount of formulae only. So we will focus on concept with real life examples and once we are good with concept, we can reproduce/ generate formula at any point of time [even you forget during exam]

### Real life example for Problems on Time and Distance:

*Let us assume you are driving bike at 40 km/hr*

- In 1 hour you will go 40 km distance
- In 2 hours you will go  $40 + 40 = 80$  Km distance
- In 3 hours you will go  $40 + 40 + 40 = 120$  Km distance

As you can observe, as number of hours increasing, covered distance is increasing by your speed of bike

So, if someone asks you how much distance you cover in 5 hours?

You will say:

If I drive bike at speed of 40 km/hr then I will cover  $40 * 5 = 200$  Km

If I drive back at speed of 50 km/hr then I will cover  $50 * 5 = 250$  Km

We can simply write as :

$$\text{Distance} = \text{Speed} * \text{Time}$$

As you are able to understand formula, you never need to mug it up. Instead just try to remember concept, automatically you will be able to recall.



**Distance = Speed \* Time**

As per requirement, you can derive other 2 formulae:

**Speed =  $\frac{\text{Distance}}{\text{Time}}$**

**Time =  $\frac{\text{Distance}}{\text{Speed}}$**

With this simple understanding our most of the problems for Time and Distance will be solved

Now we need to look at some addition terms

| Term               | Symbol        |
|--------------------|---------------|
| Meter              | m             |
| Second             | s             |
| Hour               | hr            |
| Kilometer          | km            |
| Kilometer per hour | km/hr or kmph |

| Term               | Meaning       |
|--------------------|---------------|
| 1 hour             | 60 minutes    |
| 1 minute           | 60 seconds    |
| 1 hour             | 3600 seconds  |
| 1 kilometer        | 1000 meters   |
| Kilometer per hour | Km/hr or kmph |

**Salman and Shahrukh** were fighting with each other who can run faster and drive bicycle faster.

Salman was saying he can run at speed of **18 km/hr** and he can drive bicycle at speed of **36 km/hr**.

Shahrukh said he can run at speed of **5 m/s** and drive bicycle at **10m/s**.

Deepika was listening to them and she thought of stopping this fight.

She came and said, lets sort this out step by step and she will decide who is faster among Salman and Shahrukh

### **Part 1: Running Speed**

Deepika asked Salman what is your running speed?

Salman said 18 km/hr.

Deepika said

1 km means 1000 m and 1 hr means 60 mins that is  $60 \times 60 = 3600$  seconds

So km/hr means  $1000/3600 = 10/36 = 5/18$  m/s

$18 \times 5/18 = 5$  m/s

18 km nothing but **5 m/s** which is nothing but speed of Shahrukh

So at Running speed of Salman and Shahrukh both same

### **Part 2: Bicycle Speed**

Deepika asked Salman what is your speed when you ride bicycle?

Salman said **36 km/hr**

Deepika said

1 km means 1000 m and 1 hr means 60 mins that is  $60 \times 60 = 3600$  seconds

So km/hr means  $\frac{1000 \cdot 3600}{3600} = 10/36 = 5/18$  m/s

$36 \times 5/18 = 10$  m/s

36 km/hr is nothing but **10 m/s** which is same as Bicycle speed of Shahrukh

So at Bicycle ride speed of Salman and Shahrukh both same

With this conversation we can figure out

**Conversion Table 1: Try to remember**

| Speed in Km/hr | Speed in m/s |
|----------------|--------------|
| 18             | 5            |
| 36             | 10           |
| 54             | 15           |
| 72             | 20           |
| 90             | 25           |

You can keep in mind  $18 \text{ km/hr} = 5 \text{ m/s}$  or  $5 \text{ m/s} = 18 \text{ km/hr}$  and then convert

OR

You can convert with normal formula

$$1 \text{ km/hr} = 1000/3600 = 5/18 \text{ m/s}$$

**So if you multiply any km/hr speed by 5/18 you will get speed in m/s**

For m/s to km/hr just reverse multiplication factor that is **18/5**

$$1 \text{ m/s} = 18/5 \text{ km/hr} = 3.6 \text{ km/hr}$$

$$\text{So } 1 \text{ m} = 3.6 \text{ km/hr}$$

$$2 \text{ m} = 3.6 * 2 = 7.2 \text{ km/hr}, 5 \text{ m/s} = 5 * 3.6 = 18 \text{ km/hr}$$

**Conversion Table 2: Try to remember**

| Speed in m/s | Speed in Km/hr |
|--------------|----------------|
| 1            | 3.6            |
| 2            | 7.2            |
| 5            | 18             |
| 10           | 36             |
| 20           | 72             |

**Note:**

It would be based on problem you need to see you feel multiplying by 18/5 is easy or multiplying by 3.6 is easy.

**Conversion formulae:**

**Km/hr  $\rightarrow$  m/s [ Multiply by 5/18]**

**m/s  $\rightarrow$  Km/hr [ Multiply by 18/5 or Multiply by 3.6]**

**Problem 1:**

How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr?

**Solution :**

Speed in km/hr = 20 km/hr

Time for 400 m distance = ?

As you can see here, we are supposed to find time for 400 m. However, speed given in km/hr

So before solving this we need to find speed in m/s

$$20 \text{ km/hr} = 20 * \frac{5}{18} = \frac{100}{18} \text{ m/s}$$

Now we need to find time required to cover 400 m

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{400}{100/18} = \frac{400 * 18}{100} = 4 * 18 = 72 = 1 \text{ min } 12 \text{ sec} = 1 \frac{1}{5} \text{ mins}$$

[note 12 sec = 12/60 = 1/5 mins]

**Answer is To Cover distance of 400 meters, Aditya will need  $1 \frac{1}{5}$  mins**

**Problem 2:**

A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?

**Solution :**

750 m distance in 2 min 30 sec

Speed in km/hr = ?

$$2 \text{ min } 30 \text{ sec} = (2 * 60) + 30 = 150 \text{ sec}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed} = \frac{750}{150} = 5 \text{ m/s}$$

Speed in m/s is 5 m/s

From **Conversion-Table 1** we know that 18 km = 5 m/s

Or

Just convert m/s to km/hr

$$5 * \frac{18}{5} = 18 \text{ km/hr}$$

**Answer is Speed in km/hr is 18 km/hr**

**Problem 3:**

A person crosses a 600 m long street in 5 minutes. What is speed in km/hr?

**Solution :**

600 m distance in 5 minutes

Speed in km/hr = ?

5 minutes =  $5 * 60 = 300$  s

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed} = \frac{600}{300} = 2 \text{ m/s}$$

Speed in m/s is 2 m/s

From **Conversion-Table 2** we know that  $1 \text{ m/s} = 3.6 \text{ km/hr}$

So  $2 \text{ m/s} = 2 \times 3.6 = 7.2 \text{ km/hr}$

Or

Just convert m/s to km/hr

$$2 \times \frac{18}{5} = \frac{36}{5} = 7.2 \text{ km/hr}$$

**Answer is Speed of person in km/hr is 18 km/hr**

#### **Problem 4:**

A car is running at 108 kmph. What distance will it cover in 15 seconds?

**Solution :**

Speed of Car in km/hr = 108 kmph

Distance covered in 15 s = ?

As we can see here speed is given in km/hr and distance covered in 15 seconds is asked

So we will first convert km/hr to m/s and then we can solve further

From **Conversion-Table 1** we know that  $18 \text{ km/hr} = 5 \text{ m/s}$

$36 \text{ km/hr} = 10 \text{ m/s}$  etc...

**Quick Method to convert:**

|          |     |
|----------|-----|
| Km/hr    | m/s |
| 18-----  | 5   |
| 108----- | ?   |

If we cross multiply,

$$18 \times ? = 5 \times 108$$

$$\text{Speed in m/s} = \frac{108 * 5}{18} = 6 * 5 = 30$$

**Conventional Method to convert:**

If you want to follow conventional method for conversion

$$108 * \frac{5}{18} = 30 \text{ m/s}$$

As speed of Car is 30 m/s, in 15 seconds

It will cover  $30 * 15 = 450$  m distance

**Answer is Car will cover 450 m distance in 15 seconds**

**Problem 5:**

Which of the following train is the fastest?

a) 25 m/s b) 1500 m/min c) 90 km/hr d) None of these

**Solution :**

Whenever we have to compare 2 or more different speed, distance etc., it is always preferable to convert them into single unit either km/hr or m/s

Here we have 1 with m/s , m/min and km/hr

We will convert all of them into m/s

a) 25 m/s  $\rightarrow$  25 m/s [ no conversion needed]

b) 1500 m/min

1 min = 60 seconds

So it covers 1500 m in 60 seconds

Speed in m/s =  $1500/60 = 500/20 = 25$  m/s

**Converted speed 25 m/s**

c) 90 km/hr

We know that 18 km/hr is 5 m/s

18-----5  
90-----?

Either you can cross multiply and get answer or

18-----5  
90 [= 18 \* 5]-----5 \* 5 = 25  
**Converted speed 25 m/s**

As you can see after conversion, speed is same in all a, b and c option

**Answer is None of these as all speeds are equal**

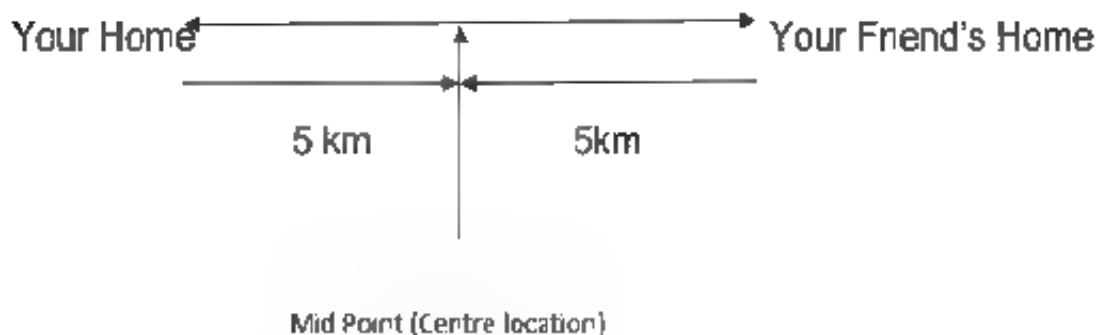
### 2 Person / Trains cross Concept:

Before solving this problem we will understand this concept with some real life example

Imagine distance between your home and your friend's home 10 km



Now both of you side to meet on exact mid location today

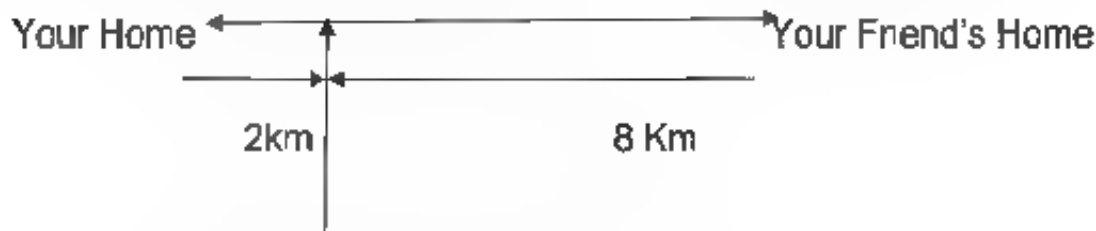


So you need to travel 5 km and your friend also need to travel so that you can meet.

Your travel + Your friend's travel = 5 + 5 = 10 km [Total distance]



Next day again you decide to meet but you did not had bike so you can travel only 2 km. But your friend has bike so he said come 2km from your house and I will meet you there



So distance traveled by you = 2km

Your friend needs to travel = 8km

Your travel + your friend's travel = total distance = 10km

### **Trick:**

*You need to remember when we are looking for meeting 2 things travelling towards each other then we need to remember*

***The point they will meet each other.***

***Dist Traveled by 1<sup>st</sup> + Distance traveled by 2<sup>nd</sup> = Total distance***

*In our case  $5 + 5 = 10$  and  $2 + 8 = 10$  etc.*

**Problem 6:**

A and B are two stations 390 km apart. A train starts from A at 10 am and travel towards B at 65 kmph. Another train starts from B at 11 am and travels towards A at 35 kmph. At what time do they meet?

**Solution :**

Distance between 2 stations = 390 Km

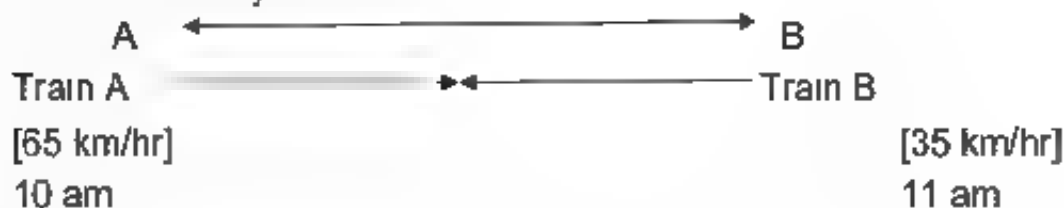
A train started from A at 10 am

Speed of Train A = 65 km/hr

B Train started from B at 11 am

Speed of train B = 35 km/hr

Time when they meet = ?



Let us assume that they meet each after x hours of 10 am

We know that at meet point.

***Dist Traveled by 1<sup>st</sup> + Distance traveled by 2<sup>nd</sup> = Total distance***

In x hours Train A will cover  $65 * x = 65x$  Km

As Train B started 1 hour after 10 am so it will be x-1 hours travel for B

Distance traveled by B would be  $35 * (x-1) = 35x - 35$

***Dist Traveled by Train A + Distance traveled by Train B = Total distance***

$$65x + 35x - 35 = 390$$

$$100x = 425$$

$$x = 4.25 \text{ Hours which is nothing but 4 hours and } \frac{1}{4} \text{ hrs}$$

$$x = 4 \text{ hours } 15 \text{ mins}$$

$$\text{So Time of Meet} = 10 \text{ am} + 4 \text{ Hours } 15 \text{ mins} = 2 \text{ pm } 15 \text{ mins}$$

**Answer is Both trains will meet at 2 :15 pm**

**Problem 7:**

A train M leaves Meerut at 5 am and reaches Delhi at 9 am. Another train leaves Delhi at 7 am and reaches Meerut at 10:30 am. At what time do the 2 trains cross each other?

**Solution :**

Train M from Meerut [5 am ] to Delhi [9 am] -> 4 hours journey

Train D from Delhi [7 am ] to Meerut [10.30 am] -> 3.5 hours journey

Time when they cross = ?

Here as you can see we don't know distance as well as we don't know speed of trains.

Our distance will be constant but speed will vary as both of train requires different time

Let us try to take some common number as distance which is divisible by 3.5 and 4

Convert decimal to normal number and then take common number so calculation would be easy

$$3.5 * 10 = 35 \text{ and } 35 * 4 = 140$$

140 is divisible by 3.5 as well as 4

So Let us assume distance = 140 km

Speed of Train M

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{140}{4} = 35 \text{ km/hr}$$

Speed of Train D

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{140}{3.5} = \frac{1400}{35} = \frac{200}{5} = 40 \text{ km/hr}$$

Let us assume that they meet each after x hours of 5 am.

We know that at meet point.

***Dist Traveled by 1<sup>st</sup> + Distance traveled by 2<sup>nd</sup> = Total distance***

In x hours Train M will cover  $35 * x = 35x$  Km

As Train D started 2 hour after 5 am so it will be x-2 hours travel for Train D

Distance traveled by D would be  $40 * (x-2) = 40x - 80$

***Dist Traveled by Train M + Distance traveled by Train D = Total distance***

$$35x + 40x - 80 = 140$$

$$75x = 220$$

$$x = \frac{220}{75} = \frac{44}{15} = 2\frac{14}{15}$$

2 hours and 14/15 hours

$$\frac{14}{15} * 60 = 56 \text{ mins}$$

$x = 2 \text{ hours } 56 \text{ Mins}$

After 2 hours and 56 Mins train will meet

5 am + 2 hours 56 mins = 7 Hours 56 mins

**Answer is Both trains will cross each other at 7:56 am**

### **Problem 8:**

A truck covers a distance of 550 meters in 1 minute whereas bus covers a distance of 33 kms in 45 mins. Ratio of their speed is ?

**Solution :**

Truck cover 550 m in 1 minute

Bus cover 33 km in 45 mins

As you can see both speed are given in different units, we need to convert them to same unit to find ratio

Let us convert both of them into m/s

**Truck:** 550 m in 1 minute

1 minute = 60 sec

Truck Speed in m/s =  $550 / 60 = 55/6 \text{ m/s}$

Bus : 33 km in 45 mins

First let us convert it into km/hr and then m/s As 1 hour = 60 mins

|    |  |      |
|----|--|------|
| Km |  | Mins |
|----|--|------|

|         |    |
|---------|----|
| 33----- | 45 |
|---------|----|

|         |    |
|---------|----|
| ? ----- | 60 |
|---------|----|

Cross multiply to get Km/hr speed

$$33 * 60 = 45 * \text{km/hr speed}$$

$$\text{Bus km/hr speed} = \frac{33 * 60}{45} = \frac{33 * 4}{3} = 44 \text{ km/hr}$$

In order to convert km/hr  $\rightarrow$  m/s , multiply by 5/18

$$\text{Bus m/s speed} = 44 * \frac{5}{18} = \frac{22 * 5}{9} = \frac{110}{9} \text{ m/s}$$

Ratio of Truck speed : Bus speed

$$55 / 6 : 110 / 9$$

$$\frac{55/6}{110/9} = \frac{55}{6} * \frac{9}{110} = \frac{55 * 9}{6 * 110} = \frac{9}{6 * 2} = \frac{9}{12} = \frac{3}{4}$$

**Answer is Ratio of Speed of Truck to Bus is 3 : 4**

#### **Problem 9:**

Three persons are walking from a Place A to another place B. Their speed are in ratio 4:3:5 The time ratio to reach by these persons will be?

**Solution :**

Speed ratio : 4 : 3 : 5

Time ratio : ?

Let us solve this problem by 2 methods 1) Concept 2) Formula

#### **Method 1: Concept oriented**

In order to find time ratio we should know how much time individual person need

We have speed given SO we will take LCM and assume it as our distance

So once we get distance and speed we can calculate time and

subsequently their ratio

LCM for 4, 3, 5 is 60 So Distance 60

Assume person A B C speed ratio is 4 : 3 : 5

So A will need =  $60 / 4 = 15$  units [min/hr anything]

B will need =  $60 / 3 = 20$  units [mins/hr anything]

C will need =  $60 / 5 = 12$  units [mins/hrs anything]

**Answer is Ratio of Time for A:B:C = 15 : 20 : 12**

**Method 2: Formula**

If ratio of Speed is given and Ratio of time to be identified or v.ice versa,  
Reverse given ratio and take ratio of it

For current Speed ratio = 4: 3: 5

$$\text{Time ratio} = \frac{1}{4} : \frac{1}{3} : \frac{1}{5}$$

Now multiply by LCM at all numerators  
= 15 · 20 · 12

**Answer is Ratio of Time for A:B:C = 15 : 20 : 12**

**Problem 10:**

Excluding stoppages, speed of bus is 54 km/hr and including stoppages, it is 45km/hr For how many minutes does bus stop per hour?

**Solution :**

Excluding stoppage speed = 54 km/hr

Considering stoppage speed = 45 km/hr

As we can see difference of distance covered when stoppage and not stoppages is

$$= 54 - 45 = 9 \text{ km}$$

In order to cover 9 km:

|         |    |
|---------|----|
| Mins    | km |
| 60----- | 54 |
| ?-----  | 9  |

Cross multiply

$$9 \text{ km time} = \frac{60 \times 9}{54} = \frac{60}{6} = 10 \text{ mins}$$

**Answer is Stoppage time is 10 mins per hour**

## Problems on Trains

Various competitive exam nations ask questions regularly based on Trains. In order to understand concept better way, please refer Time and Distance document before going through this document.

### Recap of Time and Distance document:

$$\text{Distance} = \text{Speed} * \text{Time}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

| Term               | Symbol        |
|--------------------|---------------|
| Meter              | m             |
| Second             | s             |
| Hour               | hr            |
| Kilometer          | km            |
| Kilometer per hour | km/hr or kmph |

| Term               | Meaning       |
|--------------------|---------------|
| 1 hour             | 60 minutes    |
| 1 minute           | 60 seconds    |
| 1 hour             | 3600 seconds  |
| 1 kilometer        | 1000 meters   |
| Kilometer per hour | Km/hr or kmph |

**Conversion Table 1: Try to remember**

| Speed in Km/hr | Speed in m/s |
|----------------|--------------|
| 18             | 5            |
| 36             | 10           |
| 54             | 15           |
| 72             | 20           |
| 90             | 25           |

**Conversion Table 2: Try to remember**

| Speed in m/s | Speed in Km/hr |
|--------------|----------------|
| 1            | 3.6            |
| 2            | 7.2            |
| 5            | 18             |
| 10           | 36             |
| 20           | 72             |

**Conversion formulae:****Km/hr  $\rightarrow$  m/s [ Multiply by 5/18]****m/s  $\rightarrow$  Km/hr [ Multiply by 18/5 or Multiply by 3.6]**



**Types of Problems:**

| Type                            | Given  | To Find                    |
|---------------------------------|--|----------------------------|
| Type 1(Pole Tree, Man standing) | Speed of Train, Time to cover distance, Length of Train (Any 2 values Given)                 | 3 <sup>rd</sup> value      |
| Type 2(Bridge Platform etc.)    | Length of Bridge, Length of Train, Time to cover Bridge, Speed of Train (Any 3 values Given) | 4 <sup>th</sup> value      |
| Type 3                          | 2 Trains same direction  | Speed/ Length of any Train |
| Type 4                          | 2 Trains opposite direction  | Speed/ Length of any Train |

**We will understand concept of each Type of problem before actually solving specific Type of Problem**

**Type 1: Pole, Tree, Man standing [Stationary]**

*Let say you go to beach and take 1 bucket water out of it. How much water reduced from sea? It is almost negligible right?*

*In similar way:*

*Imagine you are driving a Train Now if you have to overtake standing person or pass tree. What could be length of person or tree with respect to your Train? Do you agree it is negligible in front of your Train*

**So here you can conclude the distance cover by train to overtake person, tree or pole is same as length of Train.**

**Problem 1:**

A train 100 m long is running at speed of 30km/hr Find the time taken by it to pass man standing near the railway station

**Solution :**

Train length = 100 m

$$\text{Speed of Train} = 30 \text{ km/hr} = 30 * \frac{5}{18} = \frac{25}{3} \text{ m/s}$$

Time To pass Man standing = ?

As we know in order to pass man, train has to cover distance as her length

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{100}{25/3} = \frac{100*3}{25} = \frac{300}{25} = \frac{60}{5} = 12$$

**Answer is Time taken by Train to pass man standing near railway station is 12 seconds**

**Problem 2:**

In what time will train 100 meters long cross an electric pole if its speed be 144 km/hr?

**Solution :**

Train length = 100 m

$$\text{Speed of Train} = 144 \text{ km/hr} = 144 * \frac{5}{18} = 8 * 5 = 40 \text{ m/s}$$

Time To pass Man standing = ?

As we know in order to pass electric pole, train has to cover distance as her length

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{100}{40} = \frac{50}{20} = \frac{5}{2} = 2.5$$

**Answer is Time taken by Train to cross electric pole is 2.5 seconds**

**Problem 3:**

A train 280 m long running with speed of 63 km/hr will pass tree in?

**Solution :**

Train length = 280 m

Speed of Train = 63 km/hr =  $63 \times \frac{5}{18} = 7 \times \frac{5}{2} = \frac{35}{2}$  m/s

Time To pass Tree = ?

As we know in order to pass tree train has to cover distance as her length

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{280}{35/2} = \frac{280 \times 2}{35} = \frac{560}{35} = \frac{80}{5} = 16$$

**Answer is Time taken by Train to pass tree is 16 seconds**

**Problem 4:**

A train 132 m long passes electric pole in 6 seconds. Find the speed of train in km/hr

**Solution :**

Train length = 132 m

Time to cross pole = 6 seconds

Speed of Train = ?

As we know in order to pass electric pole, train has to cover distance as her length

And this distance is travelled by Train in 6 seconds

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed} = \frac{132}{6} = 22 \text{ m/s}$$

$$= 22 \times \frac{18}{5} = \frac{396}{5} = 79.2 \text{ km/hr}$$

**Answer is Speed of Train is 79.2 km/hr**

**Type 2: Bridge Platform etc.**

*Let us look at other real time example*

*You are driving train at Kolkata and you are passing via Howrah Bridge.  
Can you say that Bridge length is negligible as compared to Train length?*

**Obviously not** If your train length is 500 m, Bridge could be 200 m or 800 m or sometimes platform or tunnel like 1km or 2km.

*On order to cover platform or bridge, So when your train start at platform and your last boggie of train cross end of platform then you can say you completely crossed platform*

**So in these problems:**

**Distance covered by train to pass bridge or platform = Length of Train + Length of platform**

**Problem 1:**

A train moving at a speed of 132 km/hr. If length of the train is 110 meters, how long it will take to cross a railway platform 165 m long?

**Solution :**

$$\text{Speed of Train} = 132 \text{ km/hr} = 132 * \frac{5}{18} = 66 * \frac{5}{9} = 22 * \frac{5}{3} = \frac{110}{3} \text{ m/s}$$

$$\text{Length of Train} = 110 \text{ m}$$

$$\text{Railway platform length} = 165 \text{ m}$$

In order to cross railway platform Train has to cover its distance + Length of Platform

$$\text{Distance to cover} = 110 \text{ m} + 165 \text{ m} = 275 \text{ m}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{275}{110/3} = \frac{275 \times 3}{110} = \frac{55 \times 3}{22} = \frac{5 \times 3}{2} = \frac{15}{2} = 7.5$$

**Answer is Train will cross platform of 165m length in 7.5 seconds**

**Problem 2:**

How long does a train 110 meters long running at speed of 72 km/hr take to cross a bridge of 132 meters in length?

**Solution :**

$$\text{Speed of Train} = 72 \text{ km/hr} = 72 \times \frac{5}{18} = 4 \times 5 = 20 \text{ m/s}$$

$$\text{Length of Train} = 110 \text{ m}$$

$$\text{Bridge length} = 132 \text{ m}$$

In order to Bridge Train has to cover its distance + Length of Bridge

$$\text{Distance to cover} = 110 \text{ m} + 132 \text{ m} = 242 \text{ m}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{242}{20} = \frac{121}{10} = 12.1$$

**Answer is Train will cross Bridge of 132 m length in 12.1 seconds**

**Problem 3:**

A goods train running at speed of 72 km/hr and crosses 250 m long platform in 26 seconds. What is length of goods train?

**Solution :**

$$\text{Speed of Train} = 72 \text{ km/hr} = 72 \times \frac{5}{18} = 4 \times 5 = 20 \text{ m/s}$$

Platform length = 250 m and it is crossed by Goods Train in 26 seconds

Length of Train = ?

In order to cross platform Train has to cover its length + Length of Platform

Let us assume Train length as  $x$  meters

Distance covered  $= x + 250$

$x + 250$  meters distance covered in 26 seconds.

As Train speed is 20 m/s .

In 26 seconds it will cover Distance = Speed \* Time

$$= 20 * 26 = 520 \text{ m}$$

We know that distance covered in 26 seconds is  $x + 250$

So  $x + 250 = 520$

$x = 520 - 250 = 270 \text{ m}$

**Answer is Length of Goods Train is 270 m**

**Problem 4:**

A train 800 meters long running at speed of 78 km/hr If it crosses a tunnel in 1 minute then length of tunnel in meters is?

**Solution :**

Speed of Train  $= 78 \text{ km/hr} = 78 * \frac{5}{18} = 13 * \frac{5}{3} = \frac{65}{3} \text{ m/s}$

Length of Train  $= 800 \text{ m}$

Crosses Tunnel in 1 minute [60 seconds]

In order to cross Tunnel, Train has to cover its length + Length of Tunnel

Let us assume Tunnel length as  $x$  meters

Distance covered  $= 800 + x$

$800 + x$  meters distance covered in 60 seconds

As Train speed is  $\frac{65}{3} \text{ m/s}$

In 60 seconds it will cover Distance = Speed \* Time

$$= \frac{65}{3} * 60 = 65 * 20 = 1300 \text{ m}$$

We know that distance covered in 60 seconds is  $800 + x$

$$\text{So } 800 + x = 1300$$

$$x = 1300 - 800 = 500 \text{ m}$$

**Answer is Length of Tunnel is 500 m**

### **Relative Speed Concept.**

Imagine, have you ever seating in train and observing other train going for same direction or from opposite direction?

You would have always seen from window of train that who is coming from opposite side is travelling faster than our train

And if you see other train which is going at same direction as your train, you will feel next train slow even though its speed greater than you

Why it happens? It is because of relative speed

When 2 trains travels in opposite direction they cross other very fast and when they are in same direction, crossing happens very slowly

**When Train traveling from opposite direction,  
Relative speed = Speed of Train1 + Speed of Train2**

**When both Trains are travelling in same direction,  
Relative speed = Difference of 2 Trains speeds**

**If Speed of Train2 > Speed of Train1 then -> Speed of Train2 – Speed of Train1**

**If Speed of Train1 > Speed of Train2 then -> Speed of Train1 – Speed of Train2**

**Type 3: 2 Trains same direction****Problem 1:**

Two trains 100 meters and 120 meters long are running in the same direction with speed of 72km/hr and 54 km/hr. In how much time 1st train will cross second?

**Solution :**

Train1 Length = 100 m

Train2 Length = 120 m

Train 1 Speed = 72 km/hr =  $72 \times \frac{5}{18} = 4 \times 5 = 20 \text{ m/s}$

Train 2 Speed = 54 km/hr =  $54 \times \frac{5}{18} = 3 \times 5 = 15 \text{ m/s}$

*Train1 and Train2 travelling in same direction*

How much time Train1 will cross Train2 = ?

As we know train travelling in same direction ,

Relative Speed for Train1 = Train1 Speed – Train2  
 $= 20 - 15 = 5 \text{ m/s}$

For Train1, In order to cross Train 2, it has to cover Train1 Length + Train2 Length

Distance to be covered = 100 + 120 = 220 m

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{220}{5} = 44 \text{ seconds}$$

**Answer is Train1 will cross Train2 in 44 seconds**



**Problem 2:**

Two trains of equal length running on parallel lines in the same direction at 46 km/hr and 36 km/hr. The faster train passes slower train in 36 seconds. The length of each train is?

**Solution :**

Train1 Length = Train2 Length

$$\text{Train 1 Speed} = 46 \text{ km/hr} = 46 * \frac{5}{18} = \frac{23 * 5}{9} = \frac{115}{9} \text{ m/s}$$

$$\text{Train 2 Speed} = 36 \text{ km/hr} = 36 * \frac{5}{18} = \frac{18 * 5}{9} = \frac{90}{9} \text{ m/s}$$

As Train1 Speed > Train2 speed [46 > 36] . Hence Train1 is faster  
Train2 is slower

Faster train [Train1] passes slower Train[Train2] in 36 seconds

Length of Trains =?

Let us assume Length of Trains as x meters

Train1 Length = Train2 Length = x meters

For Train1, in order to pass Train2 need to cover = Length of Train1 +  
Length of Train2

Distance to be covered by Train1 to pass Train2 = x + x = 2x meters

Relative speed = Speed of Train1 – Speed of Train2

$$= \frac{115}{9} - \frac{90}{9} = \frac{25}{9} \text{ m/s}$$

Train1 Passed Train2 in 36 seconds

Distance = Speed \* Time

$$= \frac{25}{9} * 36 = 25 * 4 = 100 \text{ m}$$

We know that distance covered is 2x

So 2x = 100 m

x = 50 m

**Answer is Length of Train1 and Train2 is 50 m**

**Type 4: 2 Trains opposite direction****Problem 1:**

A Train 108 m long moving at speed of 50 km/hr crosses a train 112 m long coming from opposite direction in 6 seconds. The speed of Second Train is?

**Solution:**

Train1 Length = 108 m

Train2 Length = 112 m

$$\text{Train 1 Speed} = 50 \text{ km/hr} = 50 \times \frac{5}{18} = \frac{25 \times 5}{9} = \frac{125}{9} \text{ m/s}$$

*Train1 and Train2 travelling in opposite direction*

Train1 pass Train2 in 6 seconds

Train 2 Speed = ?

Let us assume speed of Train2 as x m/s

As we know train travelling in opposite direction ,

$$\begin{aligned} \text{Relative Speed for Train1} &= \text{Train1 Speed} + \text{Train2} \\ &= \frac{125}{9} + x \text{ m/s} \end{aligned}$$

For Train1, In order to cross Train 2, it has cover Train1 Length + Train2 Length

$$\text{Distance to be covered} = 108 + 112 = 220 \text{ m}$$

This distance is covered in 6 seconds

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed} = \frac{220}{6} \text{ m/s}$$

We know that Relative speed is  $\frac{125}{9} + x$  m/s

$$\text{So } \frac{125}{9} + x = \frac{220}{6}$$

$$x = \frac{220}{6} - \frac{125}{9}$$

$$= \frac{(220 \times 3) - (125 \times 2)}{18} = \frac{660 - 250}{18} = \frac{410}{18} \text{ m/s}$$

$$\frac{410}{18} \times \frac{18}{5} = 82 \text{ km/hr}$$

**Answer is Train2 speed is 82 km/hr**

**Problem 2:**

Two trains each 100 m long, moving in opposite directions crosses each other in 8 seconds. If one is moving as twice as fast as other, then speed of Faster train is?

**Solution:**

Let us Assume Train1 is faster and Train2 slower

Train1 Length = 100 m

Train2 Length = 100 m

Travelling in opposite direction.

Train1 crosses Train2 in 8seconds

Speed of Fast Train (Train1) =?

Let us assume speed of Train2 x m/s

As Faster train speed = 2 \* Speed of slower Train

Speed of Train1 = 2 \* Speed of Train2 = 2x m/s

Train1 to pass Train2 it needs to cover distance = Length of Train1 +  
Length of Train2

$$= 100 + 100 = 200 \text{ m}$$

As we know train travelling in opposite direction ,

$$\begin{aligned}\text{Relative Speed for Train1} &= \text{Train1 Speed} + \text{Train2 Speed} \\ &= 2x + x = 3x \text{ m/s}\end{aligned}$$

Train1 has crossed Train2 in 8 seconds

$$\begin{aligned}\text{Distance} &= \text{Speed} * \text{Time} \\ &= 3x * 8 = 24x\end{aligned}$$

We know that distance is = 200 m

$$24x = 200$$

$$x = \frac{200}{24} = \frac{100}{12} \text{ m/s}$$

$$\text{Faster Train(Train1 speed)} = 2x = 2 * \frac{100}{12} = \frac{200}{12} \text{ m/s}$$

$$\frac{200}{12} * \frac{18}{5} = \frac{40*3}{2} = 60 \text{ km/hr}$$

**Answer is Speed of Faster train is 60 km/hr**

### Boats and Streams

Various competitive examinations ask questions regularly based on Boats and Streams

Many students face challenges in understanding and solving Boats and Stream problems

Mainly we face challenge to solve Boats and Stream problems, as we never experienced much in boat travelling or rowing with/against flow of water etc. Therefore we get confused when we see terms like downstream, upstream, Speed of stream/current, still water etc.

Let us take real life example and will try to understand Boats and stream. Once you understand concept clearly you need not to take much effort to remember formula and even if you forget it during exam, with help of known concept you can generate formula as and when required.

#### Real life example to understand concept of Boats and Streams:.

Consider you have enrolled for 1<sup>st</sup> of the marathon for 1<sup>st</sup> time. Marathon means you need to complete **42km running**. As you are participating for 1<sup>st</sup> time in marathon, you started doing practice in one of playground nearby your home

Playground is completely plain surface with-out any upward or downward slope in it

After practicing for couple of month you achieved speed of running on this plain play-ground as **10 km/hr.**

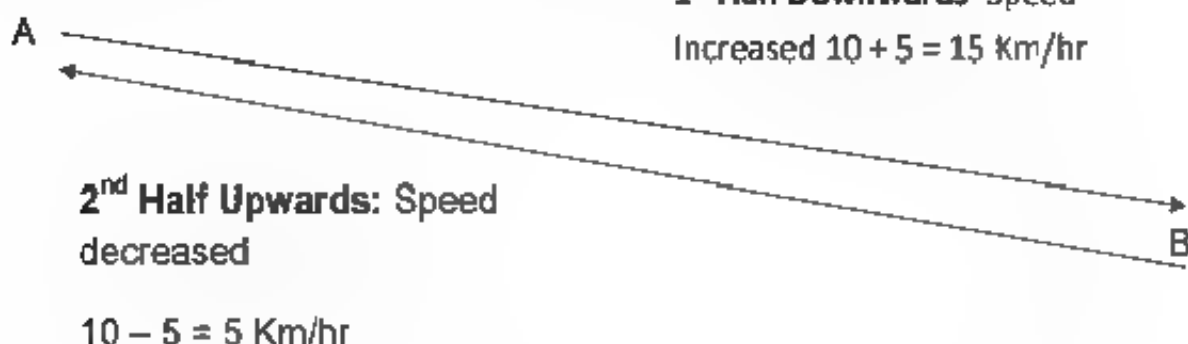
On Marathon day, you saw the Marathon track:

1<sup>st</sup> half : Start from Point A and reach to Point B [ 21 Km slope

**downwards]**

2<sup>nd</sup> Half Same route return from point B to Point A. [21 km slope **upwards]**

**Marathon start/end**



As you see in image 1<sup>st</sup> half there is complete slope downwards So obviously you get some boost for your speed and let us assume that boost is 5km/hr

While returning whatever extra boost you got in 1<sup>st</sup> half is reduced as you were coming upwards of slope and naturally your speed would reduce by 5 km/hr

Let us summarize in terms of Boats and Stream

| Real life example             | Term                                     | Speed (km/hr) |
|-------------------------------|--|---------------|
| Speed on playground           | Speed in still water                     | 10            |
| Boost you got during slope    | Speed of <b>stream / Current</b>         | 5             |
| Speed Downwards slope(A to B) | Speed at <b>Downstream / with stream</b> | $10 + 5 = 15$ |
| Speed Upwards slope(B to A)   | Speed at <b>Upstream/against stream</b>  | $10 - 5 = 5$  |

So Downstream speed will always greater than Upstream speed

Let us practice of calculating Upstream, Downstream speed and then will look at formulae and different types of problem.

| Sr No | Speed in still water(km/hr) | Speed of Stream/ Current(km/hr) | Speed at Downstream / with direction of stream(km/hr) | Speed at Upstream/against direction of stream(km/hr) |
|-------|-----------------------------|---------------------------------|---|--|
| 1     | 10                          | 3                               | $10 + 3 = 13$   | $10 - 3 = 7$   |
| 2     | 14                          | 4                               | $14 + 4 = 18$   | $14 - 4 = 10$  |
| 3     | 18                          | 5                               | $18 + 5 = 23$   | $18 - 5 = 13$  |
| 4     | 16                          | 2                               | $16 + 2 = 18$   | $16 - 2 = 14$  |
| 5     | 21                          | 6                               | $21 + 6 = 27$   | $21 - 6 = 15$  |
| 6     | 15                          | 5                               | $15 + 5 = 20$   | $15 - 5 = 10$  |

Observe this table and respective values carefully and we can get below formulae:

1) Speed at Downstream = Speed in Still water + Speed of Stream

2) Speed at Upstream = Speed in Still water – Speed of Stream

3) Speed in Still water =  $\frac{\text{Downstream speed} + \text{Upstream speed}}{2}$

4) Speed of stream =  $\frac{\text{Downstream speed} - \text{Upstream speed}}{2}$

5) Speed =  $\frac{\text{Distance}}{\text{Time}}$  and Time =  $\frac{\text{Distance}}{\text{Speed}}$

**Types of Problems:**

| Type   | Given  | To Find                                     |
|--------|--|---|
| Type 1 | Speed in still water<br>Speed of Stream/Current        | Speed at Upstream and<br>Downstream         |
| Type 2 | Speed at Upstream and<br>downstream                    | Speed in still water or speed of<br>current |
| Type 3 | Upstream, Downstream<br>distances and time<br>required | Speed in still water or speed of<br>current |
| Type 4 | Miscellaneous  |   |

**Type1:**

**Problem 1:**

Speed of boat in still water is 20 km/hr and rate of stream is 4 km/hr What is speed of boat during downstream and upstream?

**Solution :**

Speed in still water = 20 km/hr

Speed of stream = 4 km/hr

Downstream speed = ?

Upstream speed = ?

Formula:

Speed at Downstream = Speed in Still water + Speed of Stream

$$= 20 + 4$$

$$= 24 \text{ km/hr}$$

Speed at Upstream = Speed in Still water – Speed of Stream

$$= 20 - 4$$

$$= 16 \text{ km/hr}$$

**Answer is Speed at Downstream is 24 km/hr and speed at upstream is 16 km/hr**



**Problem 2:**

Speed of boat in still water is 15 km/hr and speed of stream is 1.5 km/hr  
What is speed of boat during downstream and upstream?

**Solution :**

Speed in still water = 15 km/hr

Speed of stream = 1.5 km/hr

Downstream speed = ?

Upstream speed = ?

Formula:

$$\begin{aligned}\text{Speed at Downstream} &= \text{Speed in Still water} + \text{Speed of Stream} \\ &= 15 + 1.5 \\ &= \mathbf{16.5 \text{ km/hr}}\end{aligned}$$

$$\begin{aligned}\text{Speed at Upstream} &= \text{Speed in Still water} - \text{Speed of Stream} \\ &= 15 - 1.5 \\ &= \mathbf{13.5 \text{ km/hr}}\end{aligned}$$

**Answer is Speed at Downstream is 16.5 km/hr and speed at upstream is 13.5 km/hr**

**Type2:****Problem 1:**

Speed of boat at Upstream is 7 km/hr and speed of boat at downstream is 10 km/hr. Find speed of boat in still water and rate of stream

**Solution :**

Speed at Upstream = 7 km/hr

Speed at Downstream = 10 km/hr

Speed in still water = ?

Speed of stream = ?

Formula:

$$\begin{aligned}\text{Speed in Still water} &= \frac{\text{Downstream speed} + \text{Upstream speed}}{2} \\ &= \frac{10 + 7}{2} = \frac{17}{2} \\ &= \mathbf{8.5 \text{ km/hr}}\end{aligned}$$

$$\begin{aligned}
 \text{Speed of stream} &= \frac{\text{Downstream speed} - \text{Upstream speed}}{2} \\
 &= \frac{10 - 7}{2} = \frac{3}{2} \\
 &= 1.5 \text{ km/hr}
 \end{aligned}$$

**Answer is Speed in still water is 8.5 km/hr and Speed of stream is 1.5 km/hr**

**Problem 2:**

In one hour a boat goes 11 km along the stream and 5 km against the stream. Find speed of boat in still water and rate of stream

**Solution :**

Speed at Upstream [against stream] = 5 km/hr

Speed at Downstream [along stream] = 11 km/hr

Speed in still water = ?

Speed of stream = ?

Formula:

$$\begin{aligned}
 \text{Speed in Still water} &= \frac{\text{Downstream speed} + \text{Upstream speed}}{2} \\
 &= \frac{11 + 5}{2} = \frac{16}{2} \\
 &= 8 \text{ km/hr}
 \end{aligned}$$

$$\begin{aligned}
 \text{Speed of stream} &= \frac{11 - 5}{2} \\
 &= \frac{6}{2} \\
 &= 3 \text{ km/hr}
 \end{aligned}$$

**Answer is Speed in still water is 8 km/hr and Speed of stream is 3 km/hr**

**Problem 3:**

A man can row upstream at 8 km/hr and downstream at 13km/hr. Find speed of boat in still water and rate of stream

**Solution :**

Speed at Upstream = 8 km/hr

Speed at Downstream = 13 km/hr

Speed in still water = ?

Speed of stream = ?

Formula:

$$\begin{aligned}\text{Speed in Still water} &= \frac{\text{Downstream speed} + \text{Upstream speed}}{2} \\ &= \frac{13 + 8}{2} = \frac{21}{2} \\ &= 10.5 \text{ km/hr}\end{aligned}$$

$$\begin{aligned}\text{Speed of stream} &= \frac{13 - 8}{2} \\ &= \frac{5}{2} \\ &= 2.5 \text{ km/hr}\end{aligned}$$

**Answer is Speed in still water is 10.5 km/hr and Speed of stream is 2.5 km/hr**

**Type 3:****Problem 1:**

A man takes 3 hours 45 minutes to row a boat 15 km downstream of river and 2 hours and 30 minutes to cover 5 km upstream. Find speed of the river current in km/hr

**Solution :**

*1 hour is 60 minutes so divide by 60 whenever time is given in minutes*

$$\begin{aligned}\text{Time taken to go 15 km downstream} &= 3 \text{ hours } 45 \text{ minutes} = 3 \frac{45}{60} = 3 \frac{3}{4} \\ &= \frac{15}{4} \text{ hours}\end{aligned}$$

$$\begin{aligned}\text{Time taken to go 5 km Upstream} &= 2 \text{ hours } 30 \text{ minutes} = 2 \frac{30}{60} = 2 \frac{1}{2} \\ &= \frac{5}{2} \text{ hours}\end{aligned}$$

Speed of river current = ?

We need to find downstream and upstream speed first to identify speed of river current.

Formula:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{For Downstream speed} = \frac{15}{\left(\frac{15}{4}\right)} = 4 \text{ km/hr}$$

$$\text{For Upstream speed} = \frac{5}{\left(\frac{5}{2}\right)} = 2 \text{ km/hr}$$

Now we have Upstream and downstream speed both available so we can easily find out speed of current

$$\text{Speed of stream} = \frac{\text{Downstream speed} - \text{Upstream speed}}{2}$$

$$\text{Speed of stream} = \frac{4 - 2}{2} = \frac{2}{2} = 1 \text{ km/hr}$$

**Answer is speed of river Current is 1 km/hr**

### Problem 2:

A man rows downstream 32 km and 14 km upstream. If he takes 6 hours to cover each distance, velocity (speed) of current is?

**Solution:**

Time taken to go 32 km downstream = 6 hours

Time taken to go 14 km Upstream = 6 hours

Speed of river current = ?

We need to find downstream and upstream speed first to identify speed of river current.

Formula:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{For Downstream speed} = \frac{32}{6} = \frac{16}{3} \text{ km/hr}$$

$$\text{For Upstream speed} = \frac{14}{6} = \frac{7}{3} \text{ km/hr}$$

Now we have Upstream and downstream speed both available so we can easily find out speed of current

$$\text{Speed of stream} = \frac{\text{Downstream speed} - \text{Upstream speed}}{2}$$

$$\text{Speed of stream} = \frac{\frac{16}{3} - \frac{7}{3}}{2} = \frac{\frac{9}{3}}{2} = \frac{3}{2} = 1.5 \text{ km/hr}$$

**Answer is Speed of Stream is 1.5 km/hr**

### Problem 3:

A motorboat covers certain distance downstream in 1 hour, while it comes back in  $1\frac{1}{2}$  hours. If speed of stream is 3 km/hr, what is speed of boat in still water?

**Solution :**

Let us assume speed in still water  $x$  km/hr

Downstream speed = Speed in still water + speed of current =  $(x + 3)$  km/hr

Upstream speed = Speed in still water - speed of current =  $(x - 3)$  km/hr

Distance is constant during upstream and downstream

Downstream 1 hr distance = Upstream  $1\frac{1}{2}$  hr distance

Distance = Speed \* time

During downstream distance:  $(x + 3) * 1 = x + 3$

During upstream distance =  $(x - 3) * \frac{3}{2}$

We need to solve below to get answer

$$x + 3 = (x - 3) * \frac{3}{2}$$

$$x + 3 = \frac{3x - 9}{2}$$

$$2x + 6 = 3x - 9$$

$$X = 6 + 9 = 15$$

**Answer is speed in still water is 15 km/hr**

#### **Type 4:**

##### **Problem 1:**

A motorboat whose speed is 15 km/hr in still water goes 30 km downstream and comes back in total 4 hours 30 minutes. The speed of stream in km/hr is?

##### **Solution :**

Speed in still water = 15 km/hr

Downstream 30 km + Upstream 30 km Time = 4 hours 30 minutes =  $4 \frac{30}{60} =$

$$4 \frac{1}{2} = \frac{9}{2} \text{ hours}$$

Speed of stream = ?

Let us assume speed of stream x

Downstream speed = Speed in still water + Speed of current =  $(15 + x)$  km/hr

Upstream speed = Speed in still water - Speed of current =  $(15 - x)$  km/hr

Formula:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Downstream Time} = \frac{30}{15 + x}$$

$$\text{Upstream Time} = \frac{30}{15 - x}$$

$$\frac{30}{15 + x} + \frac{30}{15 - x} = \frac{9}{2}$$

$$\frac{450 - 30x + 450 + 30x}{225 - x^2} = \frac{9}{2}$$

$$\frac{900}{225 - x^2} = \frac{9}{2}$$

$$1800 = 2025 - 9x^2$$

$$9x^2 = 2025 - 1800 = 225$$

$$x^2 = \frac{225}{9} = 25$$

Square of 5 is 25

So  $x = 5$

**Answer is speed of stream is 5 km/hr**

### Problem 2:

A man can row 5 km/hr in still water. If the velocity of current is 1 km/hr and it takes him 1 hour to row a place and come back, how far is the place?

**Solution :**

Speed in still water = 5 km/hr

Speed of current = 1 km/hr

Man rows to some place and returns back in 1 hour

First we calculate speed of Downstream and upstream

Downstream speed = Speed in still water + speed of current  
 $= 5 + 1 = 6 \text{ km/hr}$

Upstream speed = Speed in still water - speed of current  
 $= 5 - 1 = 4 \text{ km/hr}$

Let us assume total distance is  $2x$  means  $x$  distance downstream and  $x$  distance back.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Downstream Time} = \frac{x}{6}$$

$$\text{Upstream Time} = \frac{x}{4}$$

$$\frac{x}{6} + \frac{x}{4} = 1$$

$$\frac{4x+6}{24} = 1$$

$$10x = 24$$

$$x = 2.4 \text{ km}$$

**Answer is place is 2.4 km far**

**Problem 3:**

A man can row  $7\frac{1}{2}$  km/hr in still water. If river running at 1.5 km/hr, it takes him 50 minutes to row to a place and back, how far off is the place?

**Solution :**

Speed in still water =  $7\frac{1}{2}$  km/hr or 7.5 km/hr

Speed of current = 1.5 km/hr

As person going and coming back in 50 minutes

Downstream + Upstream time = 50 minutes =  $\frac{50}{60} = \frac{5}{6}$  hours

Downstream speed = speed in still water + Speed of current

$$= 7.5 + 1.5$$

$$= 9 \text{ km/hr}$$

Upstream speed = speed in still water - Speed of current

$$= 7.5 - 1.5$$

$$= 6 \text{ km/hr}$$

Let us assume total distance is 2x means x distance downstream and x distance back.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Downstream Time} = \frac{x}{9}$$

$$\text{Upstream Time} = \frac{x}{6}$$

$$\frac{x}{9} + \frac{x}{6} = \frac{5}{6}$$



$$\frac{6x+9x}{54} = \frac{5}{6}$$

$$\frac{15x}{54} = \frac{5}{6}$$

Cross multiply

$$90x = 54 \times 5$$

$$90x = 270$$

$$x = 3$$

**Answer is distance is at 3 km far**

#### Problem 4:

In stream running at 2 km/hr, motorboat goes 6 km upstream and back again to starting point 33 minutes. Find the speed of motorboat in still water.

**Solution:**

Speed of stream = 2 km/hr

$$\text{Upstream + Downstream time} = 33 \text{ minutes} = \frac{33}{60} = \frac{11}{20}$$

1 side distance is 6 km.

Let us assume speed in still water  $x$  km/hr

$$\begin{aligned} \text{Downstream speed} &= \text{speed in still water} + \text{Speed of current} \\ &= x + 2 \end{aligned}$$

$$\begin{aligned} \text{Upstream speed} &= \text{speed in still water} - \text{Speed of current} \\ &= x - 2 \end{aligned}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Upstream Time} = \frac{6}{x-2}$$

$$\text{Downstream Time} = \frac{6}{x+2}$$

$$\frac{6}{x-2} + \frac{6}{x+2} = \frac{11}{20}$$

$$\frac{6x+12+6x-12}{x^2-4} = \frac{11}{20}$$

$$\frac{12x}{x^2 - 4} = \frac{11}{20}$$

$$240x = 11x^2 - 44$$

$$11x^2 - 240x - 44 = 0$$

( $44 \times 11 = 484$  242 and 2 are factors)

$$11x^2 - 242x + 2x - 44 = 0$$

$$(x-22)(11x+2) = 0$$

$$x = 22$$

**Answer is Speed of boat is 22 km/hr**

## Simple Interest

Simple Interest is one of the most common topics across all competitive and academic exams

### Why you want Interest?

Let us assume one of your friend comes to you and ask for money of 10000 Rs. He will be giving you money back after 5 years

Are you really interested in giving money to him if he is just going to return you same amount 10000 Rs on y after 5 years?

Obviously not !!! You would be definitely expecting some amount more than that you given him. This **extra amount which you are expecting from friend is Interest.**

We will be focusing on Simple Interest only for now and once you get comfortable with simple interest we wil look for Compound Interest.

**Terms/ Terminologies in Simple Interest Calculation**

| Terms            | Symbol  | Meaning   |
|------------------|---------|---|
| Principal        | P       | Original amount given to / taken from someone       |
| Number of Years  | N       | Number of years for which amount given              |
| Rate of Interest | R       | Rate of Interest in %                               |
| Simple Interest  | I or SI | Simple Interest (Extra amount over original amount) |
| Amount           | A       | Original Amount + Simple Interest                   |

### Formulae:

Only 2 formulae are sufficient to solve any problem of Simple Interest.

It is just the way to use the formulae looking at given values matters

#### Formula 1:

$$\text{Simple Interest} = \frac{P \times N \times R}{100}$$

#### Formula 2:

$$A = P + SI$$

Let us understand this with simple example

**Sample Example**

If you given 10000 Rs money to your friend at rate of 10% per year with simple interest for 5 Years How much money your friend should return you after 5 years?

**Solution:**

$$P = 10000 \text{ Rs}$$

$$R = 10 \%$$

$$N = 5 \text{ Years}$$

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$\begin{aligned}\text{Simple Interest} &= \frac{10000 * 5 * 10}{100} \\ &= 100 * 50 \\ &= 5000\end{aligned}$$

**You would get 5000 Rs interest from your friend.**

To calculate total amount:

$$A = P + SI$$

$$= 10000 + 5000$$

$$= 15000$$

**Answer is your friend should return 15000 Rs to you after 5 years**

We will look at type of problems asked in Simple Interest and then we will solve problems for each type

**Note:**

|                  |         |
|------------------|---------|
| Principal        | P       |
| Number of Years  | N       |
| Rate of Interest | R       |
| Simple Interest  | I or SI |
| Amount           | A       |

| Type   | Given   | To find                                   |
|--------|---|---|
| Type 1 | Out of P, N, R, I, any 3 values are given                                     | 4 <sup>th</sup> value?                    |
| Type 2 | Out of P, N, R, I, A any 3 values given                                       | 4 <sup>th</sup> or 5 <sup>th</sup> value? |
| Type 3 | Interest / Amount given for 2 different time periods (eg 3 years and 5 years) | Rate of interest?                         |
| Type 4 | Principal split in 2 parts and given on 2 different Interest rates            | Original split values of Principal?       |
| Type 5 | Amount gets x times in n years  | Rate of interest?                         |

**Type 1:****Problem 1:**

Find Simple Interest on 8000 Rs at 5% per annum for 4 years?

**Solution :**

P = 8000 Rs

R = 5 %

N = 4 Years

$$\text{Simple Interest} = \frac{P \times N \times R}{100}$$

$$\begin{aligned}\text{Simple Interest} &= \frac{8000 \times 5 \times 4}{100} \\ &= 80 \times 5 \times 4 \\ &= 80 \times 20 \\ &= 1600\end{aligned}$$

**Answer is Simple Interest is 1600 Rs**

**Problem 2:**

Find Simple Interest on 7500 Rs at 10% per annum for 2 Years 4 Months?

**Solution :**

P = 7500 Rs

R = 10 %

N = 2 Years 4 months

$$= 2 \frac{4}{12} = 2 \frac{1}{3} \text{ Years} = \frac{7}{3} \text{ Years}$$

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$\begin{aligned}\text{Simple Interest} &= \frac{7500 * 7 * 10}{100 * 3} \\ &= 25 * 70 \\ &= 1750\end{aligned}$$

**Answer is Simple Interest is 1750 Rs**

**Problem 3:**

A sum fetched a total simple interest of 4016 25 Rs at rate of 9% in 5 years

What is the sum?

**Solution :**

SI = 4016 25 Rs

R = 9 %

N = 5 Years

P = ?

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$4016\ 25 = \frac{P * 5 * 9}{100}$$

$$P = \frac{4016\ 25 * 100}{5 * 9}$$

$$= \frac{4016\ 25 * 20}{9}$$

$$= 446\ 25 * 20$$

$$= 8925\ 00$$

$$= 8925$$

**Answer is Sum is 8925 Rs.**

**Problem 4:**

Anil has given amount of 10000 Rs to Sunil at the rate of 10% Simple Interest. After how much duration Sunil has to return money so that Anil will get Interest of 5000Rs?

**Solution**

$$P = 10000 \text{ Rs}$$

$$SI = 5000 \text{ Rs}$$

$$R = 10\%$$

$$N = ?$$

$$5000 = \frac{10000 * N * 10}{100}$$

$$N = \frac{5000 * 100}{10000 * 10} \\ = 5$$

**Answer is 5 Years**

**Type 2:****Problem 1**

A sum of 12,500 Rs amounts to 15 500 Rs in 4 years at the rate of simple interest. What is rate of Interest?

**Solution :**

$$P = 12500 \text{ Rs}$$

$$A = 15500 \text{ Rs}$$

$$N = 4 \text{ years}$$

$$R = ?$$

As you can see here, we have to find rate of interest.

In order to find rate of Interest we need to first find the Simple interest.

$$A = P + SI$$

$$15500 = 12500 + SI$$

$$SI = 3000$$

As per Simple Interest formula,

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$3000 = \frac{12500 * 4 * R}{100}$$

$$R = \frac{3000 * 100}{12500 * 4} = \frac{3000 * 4}{500 * 4} = 6$$

**Answer is Rate of Interest is 6%**

**Problem 2**

Reema took loan of 1200 Rs with Simple Interest for as many as years as the rate of Interest. If she paid 432 Rs as Interest at the end of loan period, what was rate of Interest?

**Solution:**

P = 1200 Rs

SI = 432 Rs

N = R = Let us assume it as x

R = ?

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$432 = \frac{1200 * x * x}{100}$$

$$x^2 = \frac{432 * 100}{1200}$$

$$= 36$$

$$x^2 = 36$$

$$x = 6$$

**Answer is Rate of Interest is 6%**

**Problem 3:**

The Simple Interest on sum of money will be 600 after 10 years. If the principal is tripled (3 times) after 5 years what will be total interest

**Solution:**

Before solving this question we will see the pattern for simple Interest with Simple Example

P = 100 Rs, R = 10% and we will find Interest till 5 years with Simple Interest

| Number of years<br>(N) | Total Interest | Amount         |
|------------------------|----------------|----------------|
| 1                      | 10 Rs          | 100 + 10 = 110 |
| 2                      | 20 Rs          | 100 + 20 = 120 |
| 3                      | 30 Rs          | 100 + 30 = 130 |



|   |       |                  |
|---|-------|------------------|
| 4 | 40 Rs | $100 + 40 = 140$ |
| 5 | 50 Rs | $100 + 50 = 150$ |

If you look at interest of 5 years:

10, 20, 30, 40, 50

→ Consistently increasing by 10 Rs.

So In **Simple Interest** as long as **Principal and Rate of Interest** is **constant Amount of Interest at any 1 year is constant value**

Let us solve problem now.

SI for 10 years = 600 Rs

If Principal trebled after 5 years Total Interest = ??

As 10 years interest is 600 Rs,

$$\text{One Year Interest} = \frac{600}{10} = 60$$

One Year Interest is 60 Rs

10 Years Interest = Interest of 1<sup>st</sup> 5 Years + Interest of last 5 years after Trebled Principal

As Principal is trebled after 5 years, Interest would also get trebled after 5 years

**[60 \* 3 per month]**

| 1 <sup>st</sup> 5 Year Interest | Last 5 Year Interest | Total Interest     |
|---------------------------------|----------------------|--------------------|
| $60 * 5 = 300$                  | $60 * 3 * 5 = 900$   | $300 + 900 = 1200$ |

**Answer is Total interest after 10<sup>th</sup> year is 1200 Rs**

### Type 3:

#### **Problem 1 :**

A certain sum of money amounts to 1008 Rs in 2 Years and 1164 Rs in 3 5 years Find Sum and rate of Interest?

**Solution :**

A (after 2 years) = 1008

A (after 3 5 years) = 1164

P = ?

$$R = ?$$

$$1.5 \text{ years Interest} = 1164 - 1008 = 156 \text{ Rs}$$

We will find 1 year interest

$$\begin{aligned}\text{One year interest} &= \frac{156}{1.5} = \frac{1560}{15} \\ &= 104\end{aligned}$$

One year Interest is 104

$$A = P + SI$$

$$\text{Amount after 2 years} = 1008$$

$$P = 1008 - (2 * 104)$$

$$P = 1008 - 208 = 800$$

**Sum is 800**

Now we will have to find rate of Interest (R)

$$SI \text{ for 2 years} = 2 * 104 = 208$$

$$P = 800$$

$$N = 2$$

$$R = ?$$

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$208 = \frac{800 * 2 * R}{100}$$

$$\begin{aligned}R &= \frac{208 * 100}{800 * 2} \\ &= \frac{208}{16} \\ &= 13\end{aligned}$$

**Rate of Interest is 13%**

**Answer is Sum is 800 Rs and Rate of Interest is 13%**

**Problem 2**

A certain sum of money at Simple interest amounts to 1012 n 2.5 years and to 1067.20 in 4 years. The rate of interest per annum is?

**Solution:**

$$A \text{ (after 2.5 years)} = 1012$$

$$A \text{ (after 4 years)} = 1067.20$$

$$P = ?$$

$$R = ?$$

$$1.5 \text{ years Interest} = 1067.20 - 1012 = 55.20 \text{ Rs}$$

We will find 1 year interest

$$\begin{aligned} \text{One year interest} &= \frac{55.20}{1.5} = \frac{5520}{150} \\ &= 36.8 \end{aligned}$$

One year Interest is 36.8

$$A = P + SI$$

$$\text{Amount after 4 years} = 1067.20$$

$$P = 1067.20 - (4 * 36.8)$$

$$P = 1067.20 - 147.20 = 920$$

**Sum is 920 Rs**

Now we will have to find rate of Interest (R)

$$SI \text{ for 4 years} = 4 * 36.8 = 147.2$$

$$P = 920$$

$$N = 4$$

$$R = ?$$

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$147.2 = \frac{920 * 4 * R}{100}$$

$$\begin{aligned} R &= \frac{147.20 * 100}{920 * 4} = \frac{14720}{920 * 4} = \frac{1472}{92 * 4} = \frac{368}{92} = \frac{16}{4} \\ &= 4 \end{aligned}$$

**Rate of Interest is 4%**

**Answer is Sum is 920 Rs and Rate of Interest is 4%**

**Type 4****Problem 1:**

A Sum of 1550 Rs is lent out into 2 parts 1 of part at 8% and another at 6% If the annual income is 106 Rs, find the money at each rate?

**Solution:**

Here we have 2 Principals, 2 different rate of Interest and Total interest is given

Let us assume 1 part of Principal (P1) as x

Other part of Principal(P2) = 1550 -x

R1 = 8%

R2 = 6%

SI = 106

We know total interest but not individual interest.

Total Interest = Interest of Part1 (x) + Interest of Part 2 (1550-x)

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$106 = \frac{x * 1 * 8}{100} + \frac{(1550 - x) * 1 * 6}{100}$$

$$106 = \frac{8x + 9300 - 6x}{100}$$

$$10600 = 2x + 9300$$

$$2x = 1300$$

$$x = 650$$

Principal 1 (P1) which is at 8% = 650 Rs

$$P2 = 1550 - x = 1550 - 650 = 900$$

**Answer is Sum given at 8% is 650 Rs and Sum given at 6% is 900**

**Type 5:****Problem 1:**

At what rate percent per annum will a sum of money double in 16 Years?

**Solution:**

Similar kind of question can be asked with triple, 8 time of sum etc

Here we need to understand how much interest we should get to make amount double

Assume if Principal is P.

To get sum  $2P$  we will  $2P - P = P$  Simple Interest

So,  $P = P$

$SI = P$

$N = 16$

$R = ?$

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$P = \frac{P * 16 * R}{100}$$

$$R = \frac{100}{16} = \frac{50}{8} = \frac{25}{4} = 6\frac{1}{4}$$

**Answer is To get Sum double in 16 years Rate of Interest should be  $6\frac{1}{4}\%$**

#### **Problem 2:**

In how many years a sum of money will be double itself at 12 % per annum?

#### **Solution :**

Similar kind of question can be asked with triple, 8 time of sum etc

Here we need to understand how much interest we should get to make amount double

Assume if Principal is P.

To get sum  $2P$  we will  $2P - P = P$  Simple Interest

So,  $P = P$

$SI = P$

$R = 12\%$

$N = ?$

$$P = \frac{P * 12 * N}{100}$$

$$N = \frac{100}{12} = \frac{50}{6} = \frac{25}{3} = 8\frac{1}{3} = 8 \text{ Year} + 4 \text{ months [1/3 means 4 Months]}$$

**Answer is to get Sum double with 12 % annum will be 8 Year and 4 Months**

### Compound Interest

Compound Interest is one of the most common topics across all competitive and academic exams. Many people face challenges while solving Compound Interest.

Most of student thinks that Simple interest is Simple and Compound Interest is difficult.

If you know Simple Interest you already know compound interest. There is only minute difference you need to understand and you will feel it easy.

Terms/ Terminologies in Simple Interest Calculation:

| Terms             | Symbol  | Meaning   |
|-------------------|---------|---|
| Principal         | P       | Original amount given to / taken from someone         |
| Number of Years   | N       | Number of years for which amount given                |
| Rate of Interest  | R       | Rate of Interest in %                                 |
| Simple Interest   | I or SI | Simple Interest (Extra amount over original amount)   |
| Amount            | A       | Original Amount + Simple Interest                     |
| Compound Interest | I or CI | Compound Interest (Extra amount over Original amount) |

### What is difference in Simple Interest and Compound Interest?

Let us assume you took 100 Rs from your friend at rate of 10%.

#### (Simple Interest)

Simple Interest after 1 year = 10 Rs [Amount will become 110, Principal: 100]

Simple Interest after 2 years =  $10 + 10 = 20$  Rs [Amount will become 120, Principal: 100]

Simple Interest after 3 years =  $10 + 10 + 10 = 30$  Rs [Amount will become 130, Principal: 100]

As you can see Interest is always constant for each year

This is because only Original Principal is always considered while calculating Simple Interest on any year.

Let us assume you took 100 Rs from your friend at rate of 10%.

**(Compound Interest)**

Simple Interest after 1 year = 10 Rs [Amount will become 110, Principal = 100]

[Please note here Principal is Revised to Amount]

Simple Interest for 2<sup>nd</sup> year =

$$\frac{P * N * R}{100} = \frac{110 * 1 * 10}{100}$$

= 11 Rs

Simple Interest after 2<sup>nd</sup> year = 11 + 10 = 21 Rs [Amount will become 121, Principal: 121]

Simple Interest for 3<sup>rd</sup> year =

$$\frac{P * N * R}{100} = \frac{121 * 1 * 10}{100}$$

= 12.1 Rs

Simple Interest after 3<sup>rd</sup> year = 11 + 10 + 12.1 = 33.1 Rs [Amount will become 133.1, Principal: 133.1]

**So basically in order to find compound interest we find simple interest only and we just keep Principal updating with Amount.**

Now same example we will solve directly with formula for Compound Interest

$$A = P \left[ 1 + \frac{R}{100} \right]^n$$

$$A = 100 \left[ 1 + \frac{10}{100} \right]^3 = 100 * 1.1^3$$

$$= 100 * 1.21 * 1.1$$

$$= 121 * 1.1$$

$$A = 133.1$$

$$\begin{aligned}\text{Compound Interest} &= \text{Amount} - \text{Principal} \\ &= 133.1 - 100 \\ &= 33.1\end{aligned}$$

Compound Interest will be 33.1 Rs and Amount will be 133.1 Rs

*So by chance, during exam if you forget Compound Interest formula and you need to calculate Compound Interest for 2 or 3 years, you can use Simple interest formula 2 or 3 times to get your answer*

**Formulae:****Formula 1:**

$$\text{Simple Interest} = \frac{P \times N \times R}{100}$$

**Formula 2:**

$$A = P + SI$$

**Formula 3:**

Compound Interest

$$A = P \left[1 + \frac{R}{100}\right]^n$$

**Formula 4:**

$$\text{Compound Interest (CI)} = A - P$$



We will look at type of problems asked on Compound Interest and then we will solve problems for each type

## Note:

|                   |         |
|-------------------|---------|
| Principal         | P       |
| Number of Years   | N       |
| Rate of Interest  | R       |
| Simple Interest   | I or SI |
| Amount            | A       |
| Compound Interest | I or CI |

| Type   | Given  | To find                 |
|--------|--|-------------------------|
| Type 1 | P, N, R given and interest compound annually                         | CI or A?                |
| Type 2 | P, N, R given and interest half yearly/ Quarterly                    | CI or A?                |
| Type 3 | Any 3 values out of 5 [P, N, R, CI, A] given                         | Any of other 2?         |
| Type 4 | Amount given for 2 different period                                  | Original Sum/Principal? |
| Type 5 | Simple Interest / compound interest or difference between them given | P or R or SI or CI?     |

## Type 1:

### **Problem 1:**

Find Compound interest on 7500 Rs at 4% per annum for 2 years, compounded annually?

### **Solution :**

P = 7500 Rs

R = 4 %

N = 2 Years

$$A = P \left[ 1 + \frac{R}{100} \right]^n$$

$$\begin{aligned}
 A &= 7500 \left[1 + \frac{4}{100}\right]^2 \\
 &\approx 7500 * \left[\frac{100+4}{100}\right]^2 \\
 &= 7500 * \left[\frac{104}{100}\right]^2 \\
 &= 7500 * \frac{26}{25} * \frac{26}{25} \\
 &= \frac{300 * 26 * 26}{25} \\
 &= 12 * 676 = 8112 \\
 \text{Amount} &= 8112 \text{ Rs}
 \end{aligned}$$

$$\begin{aligned}
 \text{CI} &= A - P \\
 \text{CI} &= 8112 - 7500 \\
 &= 612 \text{ Rs}
 \end{aligned}$$

**Answer is Compound Interest 612 Rs and Amount will be 8112 Rs**

**Problem 2:**

Amrut invested amount of 8000 Rs on fixed deposit scheme for 2 years at compound interest rate 5 p.c.p.a. How much Compound Interest and total amount Amrut will get at maturity of fixed deposit?

**Solution :**

$$\begin{aligned}
 P &= 8000 \text{ Rs} \\
 R &= 5 \% \\
 N &= 2 \text{ Years}
 \end{aligned}$$

$$\begin{aligned}
 A &= P \left[1 + \frac{R}{100}\right]^n \\
 A &= 8000 \left[1 + \frac{5}{100}\right]^2
 \end{aligned}$$

$$= 8000 * \left[ \frac{100+5}{100} \right]^2$$

$$= 8000 * \left[ \frac{105}{100} \right]^2$$

$$= 8000 * \frac{21}{20} * \frac{21}{20}$$

$$= \frac{400 * 21 * 21}{20}$$

$$= 20 * 441 = 8820$$

Amount = 8820 Rs

$$CI = A - P$$

$$CI = 8820 - 8000$$

$$= 820 \text{ Rs}$$

**Answer is Compound Interest 820 Rs and Amount will be 8820 Rs**

### Problem 3:

What will be Compound Interest and Total amount on sum of 25000 Rs after 3 years at rate of 12 p c.p. a.

**Solution :**

$$P = 25000 \text{ Rs}$$

$$R = 12 \%$$

$$N = 3 \text{ Years}$$

$$A = P \left[ 1 + \frac{R}{100} \right]^n$$

$$A = 25000 \left[ 1 + \frac{12}{100} \right]^3$$

$$= 25000 * \left[ \frac{100+12}{100} \right]^3$$

$$= 25000 * \left[ \frac{112}{100} \right]^2$$

$$= 25000 * \frac{28}{25} * \frac{28}{25} * \frac{28}{25}$$

$$\begin{aligned}
 &= \frac{1000 \times 28 \times 28 \times 28}{25 \times 25} = \frac{40 \times 28 \times 28 \times 28}{25} \\
 &= \frac{8 \times 28 \times 784}{5} \\
 &= 1.6 \times 21952 = 35123.2 \\
 \text{Amount} &= 35123.2 \text{ Rs}
 \end{aligned}$$

$$CI = A - P$$

$$\begin{aligned}
 CI &= 35123.2 - 25000 \\
 &= 10123.2 \text{ Rs}
 \end{aligned}$$

**Answer is Compound Interest 10123.2 Rs and Amount will be 35123.2 Rs**

### Type 2:

#### **Problem 1**

Find compound Interest on 10000 Rs in 2 years at 4% per annum, the interest being compounded half yearly

#### **Solution :**

Before going at actual solution of this problem we will understand difference of compound interest calculated annually, half yearly and Quarterly

| Compound Interest               | Months in Year | Number of Times in year Interest calculated |
|---------------------------------|----------------|---|
| Annually (12 Months)            | 12             | 12 / 12 = <b>1 Times</b>                    |
| Half yearly Annually (6 Months) | 12             | 12 / 6 = <b>2 Times</b>                     |
| Quarterly (3 Months)            | 12             | 12 / 3 = <b>4 Times</b>                     |

We will use same formula for Compound Interest only small changes will happen according to half yearly, quarterly etc.

**Original formula for Compound Interest (Annually):**

$$A = P \left[ 1 + \frac{R}{100} \right]^n$$

As you can imagine R means  $\frac{R}{1}$

And n means  $n * 1$

So for **Half yearly** we use **2** instead of 1 and for **Quarterly** we will use **4** instead of 1

[ Refer to above table]

**Formula for Compound Interest (Annually):**

$$A = P \left[ 1 + \frac{R}{100} \right]^n$$

**Formula for Compound Interest (Half Yearly):**

$$A = P \left[ 1 + \frac{R/2}{100} \right]^{2n}$$

**Formula for Compound Interest (Quarterly):**

$$A = P \left[ 1 + \frac{R/4}{100} \right]^{4n}$$

Now we will solve Problem

P = 10000 Rs

N = 2 years

R = 4% [ Note: Half Yearly interest calculated]

As you can see here, we have to find Compound interest and calculated half yearly.

$$\begin{aligned} A &= P \left[ 1 + \frac{R/2}{100} \right]^{2n} \\ &= 10000 \left[ 1 + \frac{4/2}{100} \right]^{(2*2)} \\ &= 10000 \left[ 1 + \frac{2}{100} \right]^{(4)} \\ &= 10000 * \left[ \frac{102}{100} \right]^4 \end{aligned}$$

$$\begin{aligned}
 &= 10000 * \frac{102}{100} * \frac{102}{100} * \frac{102}{100} * \frac{102}{100} \\
 &= \frac{102 * 102 * 102 * 102}{100 * 100}
 \end{aligned}$$

Let us do some trick here to solve  $102 * 102 * 102 * 102$

$102 * 102$  means multiply 102 by 100 and then add  $102 * 2$

$$102 * 102 = (102 * 100) + (102 * 2)$$

$$= 10200 + 204$$

$$= 10404$$

**[ I will create separate document to calculate Square in 1 step and orally as well]**

$$\text{So } 102 * 102 * 102 * 102 = 10404 * 10404$$

Let us calculate this square

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$= (10400 + 4)^2$$

$$= 10400^2 + (2 * 10400 * 4) + 4^2$$

$$= 108160000 + 83200 + 16$$

$$= 108243216$$

$$A = \frac{102 * 102 * 102 * 102}{100 * 100} = \frac{108243216}{10000}$$

$$\text{Amount} = 10824.32 \text{ Rs}$$

$$CI = A - P$$

$$= 10824.32 - 10000$$

$$CI = 824.32 \text{ Rs}$$

**Answer is Compound Interest 824.32 Rs and Amount will be 10824.32 Rs**

### Problem 2

Find compound interest on 16000 Rs at 20% per annum for 9 months.

Compound interest is calculated quarterly.

**Solution:**

$$P = 16000 \text{ Rs}$$

$R = 20\%$  [Calculated quarterly that is 4 times in year]

$N = 9 \text{ months} = \frac{9}{12} = \frac{3}{4} \text{ Years}$

$CI = ?$

$$A = P \left[1 + \frac{R/4}{100}\right]^{4n}$$

$$A = 16000 \left[1 + \frac{20/4}{100}\right]^{4 \times \frac{3}{4}}$$

$$= 16000 \left[1 + \frac{5}{100}\right]^3$$

$$= 16000 \left[\frac{105}{100}\right]^3$$

$$= 16000 * \frac{105}{100} * \frac{105}{100} * \frac{105}{100}$$

$$= \frac{16 * 105 * 105 * 105}{1000}$$

$$= \frac{16 * 21 * 21 * 105}{40} = \frac{16 * 21 * 21 * 105}{40} = \frac{16 * 21 * 21 * 21}{8}$$

$$= 2 * 21 * 441$$

$$= 42 * 441 = 18522$$

$A = 18522 \text{ Rs}$

$CI = A - P$

$$= 18522 - 16000$$

$$= 2522$$

**Answer is Compound Interest 2522 Rs and Amount will be 18522 Rs**

**Type 3:****Problem 1:**

If 500 Rs amounts to 583.20 Rs in 2 years compounded annually, find the rate of Interest per annum.

**Solution:**

$$P = 500 \text{ Rs}$$

$$N = 2 \text{ Years}$$

$$A = 583.20 \text{ Rs}$$

$$R = ?$$

$$A = P \left[1 + \frac{R}{100}\right]^n$$

$$583.20 = 500 \left[1 + \frac{R}{100}\right]^2$$

$$\frac{11664}{10000} = \left[1 + \frac{R}{100}\right]^2$$

Get square root on both sides

$$\frac{108}{100} = \left[1 + \frac{R}{100}\right]$$

$$\frac{108}{100} = \frac{100 + R}{100}$$

$$100 + R = 108$$

$$R = 8\%$$

**Answer is Rate of Interest is 8% per annum**

**Problem 2:**

In how many years will sum of 800 Rs at 10% annum compounded semi annually become 926.10 Rs?

**Solution:**

$$P = 800 \text{ Rs}$$

$$R = 10\% \text{ [ Semi/ half yearly]}$$

$$A = 926.10$$

$$N = ?$$



**As Compound Interest is Half Yearly:**

$$A = P \left[ 1 + \frac{R/2}{100} \right]^{2n}$$

$$92610 = 800 \left[ 1 + \frac{10/2}{100} \right]^{2n}$$

$$92610 = 800 \left[ 1 + \frac{5}{100} \right]^{2n}$$

$$\frac{9261}{8000} = \left[ \frac{21}{20} \right]^{2n}$$

As you can see 8000 is perfect cube of 20 and 9261 is also perfect cube of 21

$$\left[ \frac{21}{20} \right]^3 = \left[ \frac{21}{20} \right]^{2n}$$

$$2n = 3$$

$$N = 3/2$$

**Answer is 1.5 years**

### **Problem 3:**

Compound interest on 30000 Rs at 7% per annum is 4347 Rs The period in years?

**Solution:**

$$P = 30000$$

$$R = 7\%$$

$$CI = 4347 \text{ Rs}$$

$$N = ?$$

As compound Interest given we will find Amount first.

$$CI = A - P$$

$$4347 = A - 30000$$

$$A = 34347 \text{ Rs}$$

$$A = P \left[ 1 + \frac{R}{100} \right]^n$$

$$34347 = 30000 \left[1 + \frac{7}{100}\right]^n$$

$$\frac{34347}{30000} = \left[\frac{107}{100}\right]^n$$

$$\frac{11449}{10000} = \left[\frac{107}{100}\right]^n$$

$$100^2 = 10000$$

$$107^2 = 11449$$

$$n = 2 \text{ Years}$$

**Answer is 2 Years**

#### Type 4

##### Problem 1:

A sum of money amounts to 6690 after 3 years and to 1000 Rs after 6 years on compound interest. Find the sum

##### Solution

$A = 6690$  ( when  $N$  is 3 years)

$A = 10000$  (when  $A$  is 6 years)

$P = ?$

We have 2 different amounts given So we will get 2 equations with this and then we can solve them.

$$A = P \left[1 + \frac{R}{100}\right]^n$$

When  $N = 3$

$$6690 = P \left[1 + \frac{R}{100}\right]^3 \quad [\text{Equation 1}]$$

When  $N = 6$

$$10000 = P \left[1 + \frac{R}{100}\right]^6 \quad [\text{Equation 2}]$$

We will divide Equation 2 by Equation 1

$$[\text{index formula } \frac{a^m}{a^n} = a^{m-n}]$$

$$\frac{10035}{6690} = \left[1 + \frac{R}{100}\right]^3$$

$$\frac{3345}{2230} = \left[1 + \frac{R}{100}\right]^3$$

$$\frac{669}{446} = \left[1 + \frac{R}{100}\right]^3$$

$$\left[1 + \frac{R}{100}\right]^3 = \frac{3}{2}$$

Putting this value in Equation 1

$$6690 = P \times \frac{3}{2}$$

$$P = \frac{6690 \times 2}{3} = 2230 \times 2$$

$$= 4460 \text{ Rs}$$

**Answer is Sum is 4460 Rs**

### Problem 2:

A sum of money invested at compound interest amounts to Rs 800 in 3 years and to 840 Rs in 4 Years. The rate of interest is ?

**Solution:**

*We have already discussed earlier that Compound Interest is similar to Simple interest only Principal gets updated after every year.*

Here we have amount after 3 years = 800 Rs

While calculating Interest on 4<sup>th</sup> year it is same as good as simple interest on Principal after 3 years

$$\begin{aligned} \text{Simple Interest on 4}^{\text{th}} \text{ year} &= 4^{\text{th}} \text{ year amount} - 3^{\text{rd}} \text{ Year amount} \\ &= 840 - 800 \end{aligned}$$

$$\approx 40 \text{ Rs}$$

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$40 = \frac{800 * 1 * R}{100}$$

$$R = \frac{40 * 100}{800} = 5$$

**Answer is Rate of Interest is 5%**

**Type 5:**

**Problem 1:**

If the simple interest of a sum of money at 5% per annum for 3 year is 1200 Rs. Find the compound Interest on same sum for same period at same rate

**Solution:**

$$SI = 1200 \text{ Rs}$$

$$N = 3 \text{ years}$$

$$R = 5\%$$

$$CI = ?$$

As we have simple interest given we will first find Principal so that we can find compound Interest

$$\text{Simple Interest} = \frac{P * N * R}{100}$$

$$1200 = \frac{P * 3 * 5}{100}$$

$$15 P = 120000$$

$$P = \frac{120000}{15}$$

$$P = 8000 \text{ Rs}$$

$$A = P \left[ 1 + \frac{R}{100} \right]^n$$

$$A = 8000 \left[ 1 + \frac{5}{100} \right]^3$$

$$\begin{aligned}
 A &= 8000 \left[ \frac{105}{100} \right]^3 \\
 &= 8000 * \frac{105}{100} * \frac{105}{100} * \frac{105}{100} \\
 &= \frac{8*105*105*105}{1000} = \frac{8*21*21*105}{40} = \frac{21*21*105}{5} \\
 &= 21 * 21 * 21 \\
 &= \mathbf{9261 \text{ Rs}}
 \end{aligned}$$

$$CI = A - P$$

$$= 9261 - 8000$$

$$\text{Compound Interest} = 1261 \text{ Rs}$$

**Answer is Compound Interest is 1261 Rs**

### Problem 2

Difference between Compound Interest and Simple Interest on an amount of 15000 Rs for 2 years is 96Rs. What is rate of Interest?

**Solution:**

When you take any similar period and similar rate of interest and Principal, 1<sup>st</sup> Year Compound interest and simple Interest will be always same if compounded annually

2<sup>nd</sup> year onwards compound interest will be always greater than what you achieved in 2 years with simple Interest

Here we have given difference between Compound Interest and Simple Interest.

$$\text{Simple Interest} = \frac{15000 * 2 * R}{100}$$

$$\text{Simple Interest} = 300 R$$

**For Compound Interest:**

$$A = P \left[ 1 + \frac{R}{100} \right]^n$$

$$\begin{aligned}
 A &= 15000 \left[1 + \frac{R}{100}\right]^2 \\
 &= 15000 \left[\frac{100 + R}{100}\right]^2 \\
 &= 15000 \left[\frac{10000 + R^2 + 200R}{10000}\right] \\
 A &= \frac{3}{2} * [10000 + R^2 + 200R]
 \end{aligned}$$

$$\begin{aligned}
 A &= \frac{30000 + 3R^2 + 600R}{2} \\
 CI &= A - P \\
 &= \frac{30000 + 3R^2 + 600R}{2} - 15000 \\
 &= \frac{30000 + 3R^2 + 600R - 30000}{2} \\
 CI &= \frac{3R^2 + 600R}{2} \\
 CI - SI &= 96 \\
 \left[\frac{3R^2 + 600R}{2}\right] - 300R &= 96 \\
 \left[\frac{3R^2 + 600R - 600R}{2}\right] &= 96 \\
 \frac{3R^2}{2} &= 96 \\
 R^2 &= \frac{192}{3} \\
 R^2 &= 64 \\
 R &= 8
 \end{aligned}$$

**Answer is Rate of Interest is 8%**

## Clocks

Various competitive examinations ask questions regularly based on Clocks.

Clock is one of the most integral part of our day to day life

Assume you have any exam or you are going to watch movie or cricket match every thing happens and depends on Clock If you don't follow it, you will miss your bus, train or flight or probably an academic year to if you don't reach on time

### Why Clocks problems are different?

We every time look at clock to see what time it is, how much time we have before we leave home etc. However we never come across problems in real life where someone ask us what is current angle in minute hand and hour hand? What will happen if your clock loses 5 minutes every hour

Therefore, the way of us looking normally at Clock is way different than what is asked in exam Let us try to understand which things we should know before we actually solve problems on clocks

### Important formulae:

1) To find angle between minute and hour hand at any time (hh:mm)

$$30h - 5.5m$$

h- hour n timing

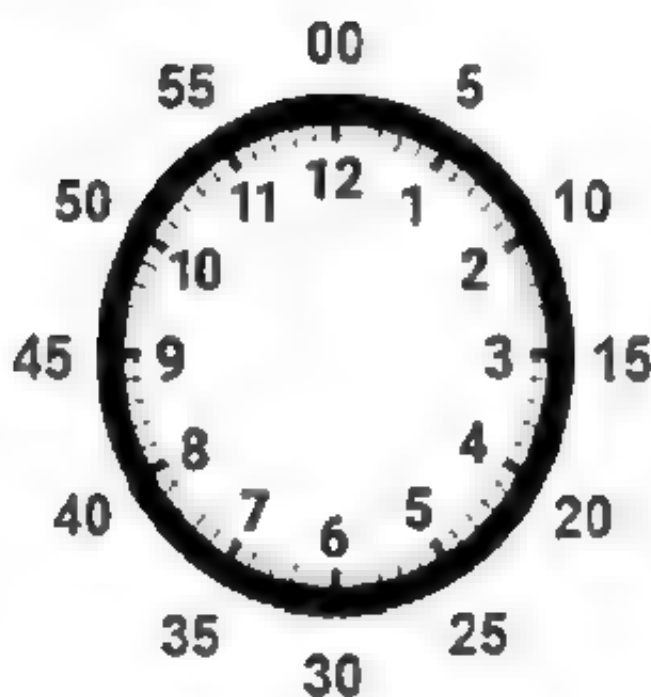
m -minute s in timing

Post calculation ignore minus sign if it comes

2) Remember fraction **12/11**

**Types of Problems:**

| Type   | To find   | Things to remember   |
|--------|---|--|
| Type 1 | What will be angle between minute hand and hour hand at <b>hh:mm</b>                          | <b>Angle = <math>30h - 5.5m</math></b><br>h – hour, m – minutes<br>Ignore sign after solving |
| Type 2 | Between 2 and 3 at what time minute hand and hour hand will <b>coincide</b>                   | 0 degree angle   |
| Type 3 | Between 3 and 4 at what time minute hand and hour hand will be <b>opposite to each other.</b> | 180 degree angle   |
| Type 4 | Between 5 and 6 at what time minute hand and hour hand will be <b>Right angle</b>             | 90 degree angle  |
| Type 5 | Static questions  |  |

**Angle Measurement in Clock**



Have look at clock and try to remember minutes along with actual digits (1 to 12)

When you see circle you can easily remember it as 360 degrees

We have total 12 numbers and it is equally spaced ( $360/12 = 30$  degrees)

**Angle between each consecutive digit on clock is 30 degrees.**

We have 5 lines between 2 consecutive numbers and so  $30/5=6$

**Angle between each consecutive minute line is 6 degrees**

Look at below 3 tables for reference and once you see you really don't need to refer it later and will be able to solve problems easily

| Angle between | Angle in degrees   | Angle between | Angle in degrees   |
|---------------|--------------------|---------------|--------------------|
| 12 and 1      | $30 \times 1 = 30$ | 6 and 7       | $30 \times 1 = 30$ |
| 1 and 2       | $30 \times 1 = 30$ | 7 and 8       | $30 \times 1 = 30$ |
| 2 and 3       | $30 \times 1 = 30$ | 8 and 9       | $30 \times 1 = 30$ |
| 3 and 4       | $30 \times 1 = 30$ | 9 and 10      | $30 \times 1 = 30$ |
| 4 and 5       | $30 \times 1 = 30$ | 10 and 11     | $30 \times 1 = 30$ |
| 5 and 6       | $30 \times 1 = 30$ | 11 and 12     | $30 \times 1 = 30$ |

**Right Angle: (90 degrees)**

| Angle between | Angle in degrees   | Angle between | Angle in degrees   |
|---------------|--------------------|---------------|--------------------|
| 12 and 3      | $30 \times 3 = 90$ | 6 and 9       | $30 \times 3 = 90$ |
| 1 and 4       | $30 \times 3 = 90$ | 7 and 10      | $30 \times 3 = 90$ |
| 2 and 5       | $30 \times 3 = 90$ | 8 and 11      | $30 \times 3 = 90$ |
| 3 and 6       | $30 \times 3 = 90$ | 9 and 12      | $30 \times 3 = 90$ |
| 4 and 7       | $30 \times 3 = 90$ | 10 and 1      | $30 \times 3 = 90$ |
| 5 and 8       | $30 \times 3 = 90$ | 11 and 2      | $30 \times 3 = 90$ |

**Opposite to each other : 180 degrees**

| Angle between | Angle in degrees    | Angle between | Angle in degrees    |
|---------------|---------------------|---------------|---------------------|
| 12 and 6      | $30 \times 6 = 180$ | 6 and 12      | $30 \times 6 = 180$ |
| 1 and 7       | $30 \times 6 = 180$ | 7 and 1       | $30 \times 6 = 180$ |
| 2 and 8       | $30 \times 6 = 180$ | 8 and 2       | $30 \times 6 = 180$ |
| 3 and 9       | $30 \times 6 = 180$ | 9 and 3       | $30 \times 6 = 180$ |
| 4 and 10      | $30 \times 6 = 180$ | 10 and 4      | $30 \times 6 = 180$ |

|          |                     |          |                     |
|----------|---------------------|----------|---------------------|
| 5 and 11 | $30 \times 6 = 180$ | 11 and 5 | $30 \times 6 = 180$ |
|----------|---------------------|----------|---------------------|

**Type1:****Problem 1:**

What will be angle between minute and hr hand at 2:30

**Solution :**

We have to remember simple formula whenever we have to calculate angle between minute n hr hand

For hh:mm =  $30h - 5.5m$  [ where h is hours and m is minutes]

$$= 30h - 5.5m$$

$$= 30 \times 2 - (5.5 \times 30)$$

$$= 60 - 165$$

$$= -105 \text{ degrees}$$

Ignore minus sign

**Answer is 105 degrees**

**Problem 2:**

What will be angle between minute and hr hand at 8:20

**Solution :**

We have to remember simple formula whenever we have to calculate angle between minute n hr hand

For hh:mm =  $30h - 5.5m$  [ where h is hours and m is minutes]

$$= 30h - 5.5m$$

$$= 30 \times 8 - (5.5 \times 20)$$

$$= 240 - 110$$

$$= 130 \text{ degrees}$$

**Answer is 130 degrees**

**Type2:****Problem 1:**

At what time between 4 to 5 minute and hour hand will coincide each other?

**Solution :**

As we have to find between 4 and 5, imagine it is 4 o'clock time



Let us make hour hand constant on 4.

In order to make minute and hour hand coincide minute hand has to travel from 12 to 4.

This is nothing but  $\rightarrow 4 \times 5 = 20$  minutes

Now recall fraction number  $12/11$  which will help to solve maximum clock problems

$$\approx 20 \times 12/11$$

$$\approx 240/11 = 21 \frac{9}{11}$$

$$= 21 \frac{9}{11}$$

$$= 4 \text{ hr} + 21 \frac{9}{11} \text{ min}$$

**Answer is 4 hr  $21 \frac{9}{11}$  mins**

**Problem 2:**

At what time between 11 to 12 minute and hour hand will coincide each other?

**Solution :**

As we have to find between 11 and 12 imagine it is 11 o'clock time



Let us make hour hand constant on 11

In order to make minute and hour hand coincide minute hand has to travel from 12 to 11

This is nothing but  $\rightarrow 11 * 5 = 55$  minutes

Now recall fraction number  $12/11$  which will help to solve maximum clock problems

$$= 55 * 12/11$$

$$= 5 * 12$$

$$= 60$$

$$= 11 \text{ hr} + 60 \text{ mins}$$

$$= 12 \text{ hr}$$

**Answer is 12 o'clock**

### **Type3:**

#### **Problem 1:**

At what time between 7 to 8 minute and hour hand will be in straight line but in opposite direction of each other?

**Solution :**

As we have to find between 7 and 8, imagine it is 7 o'clock time



Let us make hour hand constant on 7.

In order to make minute and hour hand opposite of each other:

As hr hand is on 7 and to make in straight line and opposite we need to cover 180 degrees

Opposite of 7 is 1 [you can refer table that we created earlier for 18-degrees]

So minute hand has to travel from 12 to 1

This is nothing but  $\rightarrow$  5 minutes

Now recall fraction number  $12/11$  which will help to solve maximum clock problems

$$= 5 * 12/11$$

$$= 60/11$$

$$= 5 \frac{5}{11}$$

$$= 7 \text{ hr} + 5 \frac{5}{11} \text{ mins}$$

**Answer is 7 hr  $5 \frac{5}{11}$  mins**

**Problem 2:**

At what time between 8 to 9 , minute and hour hand will be in straight line but in opposite direction of each other?

**Solution :**

As we have to find between 8 and 9, imagine it is 8 o'clock time



Let us make hour hand constant on 8.

In order to make minute and hour hand opposite of each other:

As hr hand is on 8 and to make in straight line and opposite we need to cover 180 degrees

Opposite of 8 is 2 [you can refer table that we created earlier for 180 degrees]

So minute hand has to travel from 12 to 2

This is nothing but  $\rightarrow 2 * 5 = 10$  minutes

Now recall fraction number **12/11** which will help to solve maximum clock problems

$$= 10 * 12/11$$

$$= 120/11$$

$$= 10 \frac{10}{11}$$

$$= 8 \text{ hr} + 10\frac{10}{11} \text{ mins}$$

**Answer is 8 hr  $10\frac{10}{11}$  mins**

**Type4:**

**Problem 1:**

At what time between 10 to 11, minute and hour hand will be in right angle?

**Solution :**

As we have to find between 10 and 11 imagine it is 10 o'clock time



Let us make hour hand constant on 10

In order to make minute and hour hand at right angle

As hr hand is on 10 and to make in right angle it need to travel 90 degrees

To get 90 degrees to 10 we need **to move 3 digits ahead that is 1 or 3 digits before which is 7** [you can refer table that we created earlier for 90 degrees]

So minute hand has to **travel from 12 to 1**

This is nothing but -> 5 minutes

Now recall fraction number **12/11** which will help to solve maximum clock problems

$$= 5 * 12/11$$

$$= 60/11$$

$$= 5 \frac{5}{11}$$

$$= 10 \text{ hr} + 5 \frac{5}{11} \text{ mins}$$

**And**

So minute hand has to **travel from 12 to 7**

This is nothing but  $\rightarrow$  35 minutes

Now recall fraction number **12/11** which will help to solve maximum clock problems

$$= 35 * 12/11$$

$$= 420/11$$

$$= 38 \frac{2}{11}$$

$$= 10 \text{ hr} + 38 \frac{2}{11} \text{ mins}$$

**Answer is  $10 \text{ hr } 5 \frac{5}{11} \text{ mins}$  and  $10 \text{ hr } 38 \frac{2}{11} \text{ mins}$**

**Problem 2:**

At what time between 2 to 3, minute and hour hand will be in straight line but in opposite direction of each other?

**Solution :**

As we have to find between 2 and 3, imagine it's 2 o'clock time





Let us make hour hand constant on 2.

In order to make minute and hour hand at right angle

As hr hand is on 2 and to make in right angle it need to travel 90 degrees.

To get 90 degrees to 2 we need to move 3 digits ahead that is 5 or 3 digit before which is 11 [you can refer table that we created earlier for 90 degrees]

So minute hand has to travel from 12 to 5

This is nothing but  $\rightarrow 5 * 5 = 25$  minutes

Now recall fraction number **12/11** which will help to solve maximum clock problems

$$= 25 * 12/11$$

$$= 300/11$$

$$= 27 \frac{3}{11}$$

$$= 2 \text{ hr} + 27 \frac{3}{11} \text{ mins}$$

And

So minute hand has to travel from 12 to 11

This is nothing but  $\rightarrow 11 * 5 = 55$  minutes

Now recall fraction number  $12/11$  which will help to solve maximum clock problems

$$= 55 * 12/11$$

$$= 60 \text{ minute}$$

$$= 2 \text{ hr} + 60 \text{ mins} = 3 \text{ Hr}$$

**Answer is 2 hr  $27\frac{3}{11}$  mins and 3 Hr**

**Type5:**

**Problem 1:**

How many times do hands (hour and minutes hand) of clock coincide in a day?

**Solution:**

In 12 hrs

| Timings | Number of coincide | Timings  | Number of coincide |
|---------|--------------------|----------|--------------------|
| 11 to 1 | Only 1 time        | 5 to 6   | 1                  |
|         |                    | 6 to 7   | 1                  |
| 1 to 2  | 1                  | 7 to 8   | 1                  |
| 2 to 3  | 1                  | 8 to 9   | 1                  |
| 3 to 4  | 1                  | 9 to 10  | 1                  |
| 4 to 5  | 1                  | 10 to 11 | 1                  |

In 12 hrs: 11 times

In entire day 24 hrs : 22 times

**Answer is 22 Times in a day hands of clock coincide**

**Problem 2:**

How many times do hands (hour and minutes hand) of clock are straight?

**Solution:**

Hands of clock coincides 22 times in day

[Note: Coincide 1 time between 11 to 1 and for rest 1 hr 1 time each]

Hands of clock are in opposite direction 22 times

[Note: Opposite in direction 1 time only between 5 to 7 and for rest 1 hr 1 time each]

**Answer is 44 times in day hands of clock are straight**

**Problem 3:**

How many times do hands (hour and minutes hand) of clock are in right angle?

**Solution:**

**Answer is 44 Times in a day hands of clock are in right angle**

**Problem 4:**

How many times do hands (hour and minutes hand) of clock are straight line but opposite in direction?

**Solution :**

[Note: Opposite and straight 1 time only between 5 to 7 and for rest 1 hr 1 time each]

**Answer is 22 times in day hands of clock are in straight and opposite direction.**

## Calendar

Various competitive examinations ask questions regularly based on Calendar. Calendar is the thing by which our day starts. If someone asks you what is today's date and day you will check your mobile or laptop or sometime watch if it has capability to see date and day.

How you would feel if you can calculate **day of any date of any century** irrespective of it is Past or Present or Future date???

Yes. If you go through below details and examples, able to memorize just 4 to 5 numbers and dates, you can definitely achieve this.

Initial focus would be on understanding calculation of day of Any date and once you get familiar, **you would be able to calculate day orally within 1 minute.**

Before proceeding to calculation we should know below terms thoroughly:

- 1) Leap year and Non-Leap (Normal) year
- 2) Odd days meaning
- 3) Which month contains 31 days or 30 days

**1) Leap Year:** Look at year and see it is coming under which condition

Condition 1: Any year in which **last 2 digits** are not 00

→ If year is completely divisible by 4 then it is Leap Year.

Condition 2: Any year in which **last 2 digits** are 00

→ If year is completely divisible by 400 then it is Leap Year

| Parameters         | Normal Year( NonLeap Year)                        | Leap Year   |
|--------------------|---|---|
| Number of days     | 365   | 366   |
| Number of Odd days | 1 [ when you divide 365 by 7 then remainder is 1] | 2 [ when you divide 366 by 7 then remainder is 2] |
| Date difference    | February 28 days only                             | February 29 days                                  |
|                    |   |   |

Let us look at few examples and make sure that we can identify Leap year or Non Leap Year

| Year | Divisible by 4 | Divisible by 400   | Leap / Non-Leap Year  |
|------|----------------|--|-----------------------|
| 2021 | No             | Not required as last 2 digit <b>21</b>                       | Non Leap/ Normal Year |
| 1976 | Yes            | Not required as last 2 digit <b>76</b>                       | Leap Year             |
| 1900 | Yes            | Not divisible by 400 and contains <b>00</b> at last 2 digits | Non Leap/ Normal Year |
| 1600 | Yes            | Yes and contains last 2 digits <b>00</b>                     | Leap Year             |
| 2028 | Yes            | Not required as last 2 digit <b>28</b>                       | Leap Year             |

## 2) Odd days meaning

We know each week contains 7 days.

Each normal year has 1 odd day.

Normal year contains 365 days. If we calculate  $365/7$

$$\frac{365}{7}, \text{ Quotient} = 52 \text{ and Remainder} = 1$$

This means that our normal year has complete 52 weeks and 1 odd day

**Normal Year 1 Odd day.**

1 Odd day means [getting next day]

If 10<sup>th</sup> January 2021 has Monday

→ Then 10<sup>th</sup> January 2022 will have Tuesday [Monday + 1 = Tuesday]

If 7<sup>th</sup> June 2021 has Sunday

→ Then 7<sup>th</sup> January 2022 will have Monday [Sunday + 1 = Monday]

**Leap year has 2 Odd days and**

If 10<sup>th</sup> January 2020 has Monday

→ Then 10<sup>th</sup> January 2021 will have Wednesday

If 5<sup>th</sup> June 2020 has Friday

→ Then 5<sup>th</sup> June 2021 will have Saturday not Sunday

Please Note here even though 2020 is leap year it is moving with 1 day only because this range [ 5<sup>th</sup> June 2020 to 5<sup>th</sup> June 2021 does not contain leap year February part.

If 10<sup>th</sup> June 2019 has Monday

→ Then 10<sup>th</sup> June 2020 will have Wednesday [Monday + 2 = Wednesday]

→ This is because this range [10<sup>th</sup> June 2019 to 10<sup>th</sup> June 2020 contains leap year February part]

### 3) Which month contains 31 days or 30 days?

Most of the students get confused which month contains 30 or 31 February month any one able to remember as it is different from all months So if you try to remember 30 days month so we can remember all 12 months days properly.

#### Simple Trick:

Imagine 4 people which you know [ look for people whom you are close] and they have their birthday in April, June September, November and taking 1 from each month assume them as they are single family and create 1 picture of it.

Now at any point you look for calendar problem you would know which are 30 days , 28/29 days and remaining all months with 31

You can try another trick but just remember these 4 months with 30 days

| 31 Days Month | 30 Days Month | Others (28 or 29)        |
|---------------|---------------|--------------------------|
| January       | April         | February (Leap -29)      |
| March         | June          | February (Non-Leap - 28) |
| May           | September     |                          |
| July          | November      |                          |
| August        |               |                          |
| October       |               |                          |

|          |  |  |
|----------|--|--|
| December |  |  |
|----------|--|--|

**Steps required for solving Calendar Problems****[ Step 1] – Identify odd days in century****Table 1: Century Odd days**

| Time period from start of calendar | Number of Odd days |
|------------------------------------|--------------------|
| 100 Years                          | 5                  |
| 200 Years                          | 3                  |
| 300 Years                          | 1                  |
| 400 Years                          | 0                  |

**Table 2: Yearly Odd days [ Step 2]**

| Year          | Number of Odd days |
|---------------|--------------------|
| 1 Normal Year | 1                  |
| 1 Leap Year   | 2                  |

**Table 3: Year distribution with day number Step3**

We have divided entire year into 4 equal part to make calculations simple and orally

Trick written in bracket to memorize you can use your own technique.

Normal year if you remember just 1 day to be reduced if date is post 28 Feb

| N th day of the year | Normal/Non Leap Year     | Leap Year    |
|----------------------|--------------------------|--------------|
| 1                    | 1 January [Year start]   | 1 January    |
| 90                   | 31 March [March ending]  | 30 March     |
| 180                  | 29 June                  | 28 June      |
| 270                  | 27 September             | 26 September |
| 360                  | 26 December [Boxing day] | 25 December  |

Let us take 5 different dates and calculate their day in detail and post that in just single row we will be able to calculate day

**Problem 1:**

What was the day on 15<sup>th</sup> August 1947?

**Solution:**

Let us see how we will split and calculate odd days 1 by 1 to reach till 15<sup>th</sup> August 1947 [1947 = 1900 + 46 + Current year day for 15<sup>th</sup> August]

**Step1: Calculate odd days till century [Century means nearest lower or equal century number]**

**1900 is nearest century number.**

**Odd days till 1900:**

| Year         | Odd days                        |
|--------------|---------------------------------|
| 0001 to 0400 | 0 [ as 400 years has 0 odd day] |
| 0401 to 0800 | 0 [ as 400 years has 0 odd day] |
| 0801 to 1200 | 0 [ as 400 years has 0 odd day] |
| 1201 to 1600 | 0 [ as 400 years has 0 odd day] |
| 1600 to 1900 | 1 [ as 300 years has 1 odd day] |

**Odd days till 1900 : 1**

**Step 2: Calculate odd days till end of previous year.**

As we are calculating day for 15<sup>th</sup> August 1947, our previous year would be 1946.

| Year                                     | Number of Years  |
|--|--|
| 1901 to 1946 [ Number of Leap years]     | 11   |
| 1901 to 1946 [ Number of Non Leap Years] | 35 [ calculate leap 1 <sup>st</sup> and minus in from total like 46 -11 = 35 |

We have 1 odd day for normal year and 2 odd days for Leap year

$$\begin{aligned}\text{Odd days from 1901 to 1946} &= (35 * 1) + (11 * 2) \\ &= 35 + 22 = 57\end{aligned}$$

Week is of 7 days and always it keeps repeating

So divide by 7 and calculate remainder which will give us odd day count.



$$\frac{57}{7}, \text{Quotient} = 8 \text{ and Remainder} = 1$$

**Odd days from 1901 to 1946 is 1**

**Step 3: Calculate odd days in current year till required date**

**Calculate the day number of 15<sup>th</sup> August in current year (1947).**

Here our day number table will help and it will look very simple to calculate rather than calculating month-wise. 1947 is a normal year as not divisible by 4.

| N <sup>th</sup> day of the year | Normal/Non Leap Year     |
|---------------------------------|--------------------------|
| 1                               | 1 January [Year start]   |
| 90                              | 31 March [March ending]  |
| 180                             | 29 June                  |
| 270                             | 27 September             |
| 360                             | 26 December [Boxing day] |

Find nearest date to 15<sup>th</sup> August in this table. Better to go previous date.

29<sup>th</sup> June is 180 number day

In order to get number from 29<sup>th</sup> June to 15<sup>th</sup> August

$$= 1 \text{ [June ends on 30]} + 31 \text{ [July month]} + 15$$

$$= 47$$

As 29<sup>th</sup> June is 180<sup>th</sup> day

$$180 + 47 = 227$$

To get odd days

$$\frac{227}{7}, \text{Quotient} = 32 \text{ and Remainder} = 3$$

**Number of odd days from 1<sup>st</sup> January 1947 to 15<sup>th</sup> August 1947 : 3**

Now let us sum up all step output to get our answer

| Steps         | Calculation   |                   |
|---------------|---|-------------------|
| Step 1        | Odd days Till 1900  | 1                 |
| Step 2        | Odd days from 1901 to 1946  | 1                 |
| Step 3        | Odd days from 1 <sup>st</sup> Jan to 15 <sup>th</sup> August 1947 | 3                 |
| <b>Answer</b> | <b>Total [ final remainder after dividing by 7]</b>               | <b>5 - FRIDAY</b> |

**This is really simple to remember as week starts on Monday.**

| Answer | Day       |
|--------|-----------|
| 1      | Monday    |
| 2      | Tuesday   |
| 3      | Wednesday |
| 4      | Thursday  |
| 5      | Friday    |
| 6      | Saturday  |
| 0      | Sunday    |

**Answer is Friday**

### **Problem 2:**

What was the day on 29<sup>th</sup> September 2000?

#### **Solution:**

Let us see how we will split and calculate odd days 1 by 1 to reach till 29<sup>th</sup> September 2000 [2000 = 1900 + 99 + Current year day for 29<sup>th</sup> September]

**Step1: Calculate odd days till century [Century means nearest lower or equal century number]**

**1900 is nearest century number.**

**Odd days till 1900:**

| Year         | Odd days                        |
|--------------|---------------------------------|
| 0001 to 0400 | 0 [ as 400 years has 0 odd day] |
| 0401 to 0800 | 0 [ as 400 years has 0 odd day] |

|              |                                 |
|--------------|---------------------------------|
| 0801 to 1200 | 0 [ as 400 years has 0 odd day] |
| 1201 to 1600 | 0 [ as 400 years has 0 odd day] |
| 1600 to 1900 | 1 [ as 300 years has 1 odd day] |

**Odd days till 1900 : 1**

**Step 2: Calculate odd days till end of previous year.**

As we are calculating day for 29<sup>th</sup> September 2000, our previous year would be 1999:

| Year                                     | Number of Years  |
|--|--|
| 1901 to 1999 [ Number of Leap years]     | 24   |
| 1901 to 1999 [ Number of Non Leap Years] | 75 [ calculate leap 1 <sup>st</sup> and minus in from total like 99 -24 = 75 |

We have 1 odd day for normal year and 2 odd days for Leap year

$$\begin{aligned}\text{Odd days from 1901 to 1999} &= (75 * 1) + (24 * 2) \\ &= 75 + 48 = 123\end{aligned}$$

Week is of 7 days and always it keeps repeating

So divide by 7 and calculate remainder which will give us odd day count.

$$\frac{123}{7}, \text{ Quotient} = 17 \text{ and Remainder} = 4$$

**Odd days from 1901 to 1999 is 4**

**Step 3: Calculate odd days in current year till required date**

**Calculate the day number of 29<sup>th</sup> September in current year (2000).**

Here our day number table will help and it will look very simple to calculate rather than calculating month-wise. 2000 is Leap year as divisible by 4 and 400 as well (last 2 digit 00)

| N <sup>th</sup> day of the year | Normal/Non Leap Year    | Leap Year    |
|---------------------------------|-------------------------|--------------|
| 1                               | 1 January [Year start]  | 1 January    |
| 90                              | 31 March [March ending] | 30 March     |
| 180                             | 29 June                 | 28 June      |
| 270                             | 27 September            | 26 September |
| 360                             | 26 December [Boxing     | 25 December  |

|  |      |  |
|--|------|--|
|  | day] |  |
|--|------|--|

Find nearest date to 29<sup>th</sup> September in this table. Better to go previous date.

26<sup>th</sup> September is 180 number day

In order to get number from 26<sup>th</sup> September to 29<sup>th</sup> September  
= 3

As 26<sup>th</sup> September is 270<sup>th</sup> day

$$270 + 3 = 273$$

To get odd days

$$\frac{273}{7}, \text{ Quotient} = 39 \text{ and Remainder} = 0$$

**Number of odd days from 1<sup>st</sup> January 2000 to 29<sup>th</sup> September 2000 : 0**

Now let us sum up all step output to get our answer

| Steps         | Calculation  |                   |
|---------------|--|-------------------|
| Step 1        | Odd days Till 1900   | 1                 |
| Step 2        | Odd days from 1901 to 1999   | 4                 |
| Step 3        | Odd days from 1 <sup>st</sup> Jan to 29 <sup>th</sup> September 2000 | 0                 |
| <b>Answer</b> | <b>Total [ final remainder after dividing by 7]</b>                  | <b>5 - Friday</b> |

**This is really simple to remember as week starts on Monday.**

| Answer | Day       |
|--------|-----------|
| 1      | Monday    |
| 2      | Tuesday   |
| 3      | Wednesday |
| 4      | Thursday  |
| 5      | Friday    |
| 6      | Saturday  |
| 0      | Sunday    |

**Answer is Friday**

**Problem 3:**

What will be the day on 20<sup>th</sup> November 2031?

**Solution:**

Let us see how we will split and calculate odd days 1 by 1 to reach till 20<sup>th</sup> November 2031. [2031 = 2000 + 30 + Current year day for 20<sup>th</sup> November]

**Step1: Calculate odd days till century [Century means nearest lower or equal century number]**

**2000 is nearest century number.**

**Odd days till 2000:**

| Year         | Odd days                        |
|--------------|---------------------------------|
| 0001 to 0400 | 0 [ as 400 years has 0 odd day] |
| 0401 to 0800 | 0 [ as 400 years has 0 odd day] |
| 0801 to 1200 | 0 [ as 400 years has 0 odd day] |
| 1201 to 1600 | 0 [ as 400 years has 0 odd day] |
| 1600 to 2000 | 0 [ as 400 years has 1 odd day] |

**Odd days till 2000 : 0**

**Step 2: Calculate odd days till end of previous year.**

As we are calculating day for 20<sup>th</sup> November 2031, our previous year would be 2030:

| Year                                     | Number of Years  |
|--|--|
| 2001 to 2030 [ Number of Leap years]     | 7  |
| 2001 to 2030 [ Number of Non Leap Years] | 23 [ calculate leap 1 <sup>st</sup> and minus in from total like 30 - 7 = 23 |

We have 1 odd day for normal year and 2 odd days for Leap year

$$\begin{aligned}\text{Odd days from 2001 to 2030} &= (23 * 1) + (7 * 2) \\ &= 23 + 14 = 37\end{aligned}$$

Week is of 7 days and always it keeps repeating

So divide by 7 and calculate remainder which will give us odd day count.

$$\frac{37}{7}, \text{Quotient} = 5 \text{ and Remainder} = 2$$

**Odd days from 2001 to 2030 is 2**

**Step 3: Calculate odd days in current year till required date**

**Calculate the day number of 20<sup>th</sup> November in current year (2031).**

Here our day number table will help and it will look very simple to calculate rather than calculating month-wise. 2031 is a Normal year as not divisible by 4.

| N <sup>th</sup> day of the year | Normal/Non Leap Year     |
|---------------------------------|--------------------------|
| 1                               | 1 January [Year start]   |
| 90                              | 31 March [March ending]  |
| 180                             | 29 June                  |
| <b>270</b>                      | <b>27 September</b>      |
| 360                             | 26 December [Boxing day] |

Find nearest date to 20<sup>th</sup> November in this table. Better to go previous date.  
27<sup>th</sup> September is 270 number day

In order to get number from 27<sup>th</sup> September to 20<sup>th</sup> November  
= 3 [September has 30 days] + 31 [October 31 day] + 20  
= 54

As 27<sup>th</sup> September is 270<sup>th</sup> day.

$270 + 54 = 324$

To get odd days

$$\frac{324}{7}, \text{Quotient} = 46 \text{ and Remainder} = 2$$

**Number of odd days from 1<sup>st</sup> January 2031 to 20<sup>th</sup> November 2031 : 2**

Now let us sum up all step output to get our answer

| Steps         | Calculation   |                     |
|---------------|---|---------------------|
| Step 1        | Odd days Till 2000  | 0                   |
| Step 2        | Odd days from 2001 to 2030  | 2                   |
| Step 3        | Odd days from 1 <sup>st</sup> Jan to 20 <sup>th</sup> November 2031 | 2                   |
| <b>Answer</b> | <b>Total [ final remainder after dividing by 7]</b>                 | <b>4 - Thursday</b> |

**This is really simple to remember as week starts on Monday.**

| Answer | Day       |
|--------|-----------|
| 1      | Monday    |
| 2      | Tuesday   |
| 3      | Wednesday |
| 4      | Thursday  |
| 5      | Friday    |
| 6      | Saturday  |
| 0      | Sunday    |

**Answer is Thursday**

#### **Problem 4:**

What will be the day on 18<sup>th</sup> April 1967?

#### **Solution:**

Let us see how we will split and calculate odd days 1 by 1 to reach till 18<sup>th</sup> April 1967 [1967 = 1900 + 66 + Current year day for 18<sup>th</sup> April]

**Step1: Calculate odd days till century [Century means nearest lower or equal century number]**

**1900 is nearest century number.**

**Odd days till 1900:**

| Year         | Odd days                        |
|--------------|---------------------------------|
| 0001 to 0400 | 0 [ as 400 years has 0 odd day] |
| 0401 to 0800 | 0 [ as 400 years has 0 odd day] |
| 0801 to 1200 | 0 [ as 400 years has 0 odd day] |
| 1201 to 1600 | 0 [ as 400 years has 0 odd day] |
| 1600 to 1900 | 1 [ as 300 years has 1 odd day] |

**Odd days till 1900 : 1**

**Step 2: Calculate odd days till end of previous year.**

As we are calculating day for 18<sup>th</sup> April 1967, our previous year would be **1966:**

| Year                          | Number of Years |
|-------------------------------|-----------------|
| 1901 to 1966 [ Number of Leap | 16              |

|  |  |
|--|--|
| years]                                   |  |
| 1901 to 1966 [ Number of Non Leap Years] | 50 [ calculate leap 1 <sup>st</sup> and minus in from total like 66 - 16 = 50] |

We have 1 odd day for normal year and 2 odd days for Leap year

$$\begin{aligned}\text{Odd days from 1901 to 1966} &= (50 * 1) + (16 * 2) \\ &= 50 + 32 = 82\end{aligned}$$

Week is of 7 days and always it keeps repeating

So divide by 7 and calculate remainder which will give us odd day count.

$$\frac{82}{7}, \text{Quotient} = 11 \text{ and Remainder} = 5$$

**Odd days from 1901 to 1966 is 5**

**Step 3: Calculate odd days in current year till required date**

**Calculate the day number of 19<sup>th</sup> April in current year (1967).**

Here our day number table will help and it will look very simple to calculate rather than calculating month-wise. 1967 is Normal year as not divisible by 4.

| N <sup>th</sup> day of the year | Normal/Non Leap Year     |
|---------------------------------|--------------------------|
| 1                               | 1 January [Year start]   |
| 90                              | 31 March [March ending]  |
| 180                             | 29 June                  |
| 270                             | 27 September             |
| 360                             | 26 December [Boxing day] |

Find nearest date to 18<sup>th</sup> April in this table. Better to go previous date.

31<sup>st</sup> March is 90<sup>th</sup> number day

In order to get number from 31<sup>st</sup> March to 18<sup>th</sup> April  
= 18

As 31<sup>st</sup> March is 90<sup>th</sup> day

$$90 + 18 = 108$$

To get odd days



$$\frac{108}{7}, \text{ Quotient} = 15 \text{ and Remainder} = 3$$

**Number of odd days from 1<sup>st</sup> January 1967 to 18<sup>th</sup> April 1967 : 3**

Now let us sum up all step output to get our answer

| Steps         | Calculation  |  |
|---------------|--|--|
| Step 1        | Odd days Till 1900   | 1  |
| Step 2        | Odd days from 1901 to 1966                                       | 5  |
| Step 3        | Odd days from 1 <sup>st</sup> Jan to 18 <sup>th</sup> April 1967 | 3  |
| <b>Answer</b> | <b>Total [ final remainder after dividing by 7]</b>              | <b>9 [9/7 Remainder will be 2 – Tuesday]</b> |

**This is really simple to remember as week starts on Monday.**

| Answer | Day       |
|--------|-----------|
| 1      | Monday    |
| 2      | Tuesday   |
| 3      | Wednesday |
| 4      | Thursday  |
| 5      | Friday    |
| 6      | Saturday  |
| 0      | Sunday    |

**Answer is Tuesday**

**Multiplication Shortcut Tricks**

How much time you will need to calculate answer

- 1)  $99 * 99$
- 2)  $786 * 785$
- 3)  $76 * 45$
- 4)  $689 * 546$
- 5)  $38 * 777$
- 6)  $88 * 88$

Solving any mathematics problems quickly needs calculation of Multiplication, Squares etc. More time we need to do calculations, more time we will take to solve any math problem.

What if you can calculate square or multiplications less than 20 seconds and with good practice orally as well?

We can reduce our problem solving time for each question as 90% of problem need multiplication whether it is Work and Time, Profit and Loss, Percentage, Compound Interest or any other topic.

We will see simple tricks to calculate

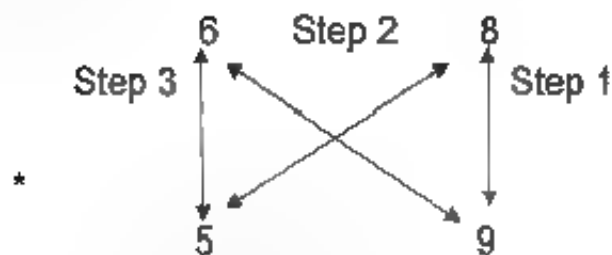
**Multiplication**

- 1) 2 digit number \* 2 digit number
- 2) 3 digit number \* 3 digit number
- 3) 2 digit number \* 3 digit number
- 4) 3 digit number \* 2 number

## 1) Type 1: 2 digit number \* 2 digit number

**Problem 1:**

$68 * 59$

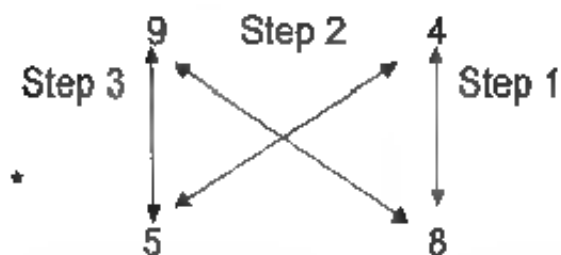
**Solution:**

| Step   | Description                           | Calculation  | Answer |
|--------|---------------------------------------|--|--------|
| Step 1 | Multiply rightmost 2 digits           | $8 * 9 = 72$ [write 2 and <b>carry 7</b> ]   | 2      |
| Step 2 | Multiply diagonal digits and add them | $(6 * 9) + (8 * 5) = 54 + 40$<br>$= 94$<br>$94 + \text{Previous carry}$<br>$94 + 7 = 101$<br>[ write 1 and <b>carry 10</b> ] | 12     |
| Step 3 | Multiply left most 2 digit            | $6 * 5 = 30$<br>$30 + \text{Previous carry}$<br>$30 + 10 = 40$   | 4012   |

**Answer is 4012**

**Problem 2:**

$94 * 58$

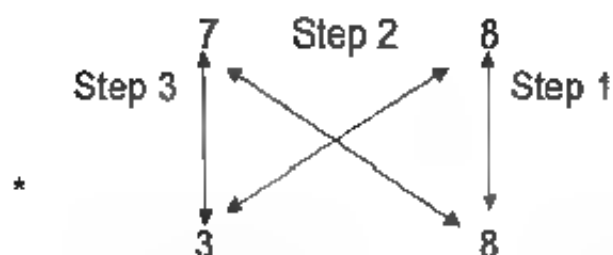
**Solution:**

| Step   | Description                           | Calculation   | Answer |
|--------|---------------------------------------|---|--------|
| Step 1 | Multiply rightmost 2 digits           | $4 * 8 = 32$<br>[write 2 and carry 3]   | 2      |
| Step 2 | Multiply diagonal digits and add them | $(9 * 8) + (4 * 5) = 72 + 20$<br>= 92<br>92 + Previous carry<br>$92 + 3 = 95$<br>[ write 5 and carry 9] | 52     |
| Step 3 | Multiply left most 2 digit            | $9 * 5 = 45$<br>45 + Previous carry<br>$45 + 9 = 54$  | 5452   |

**Answer is 5452**

**Problem 3:**

$78 * 38$

**Solution:**

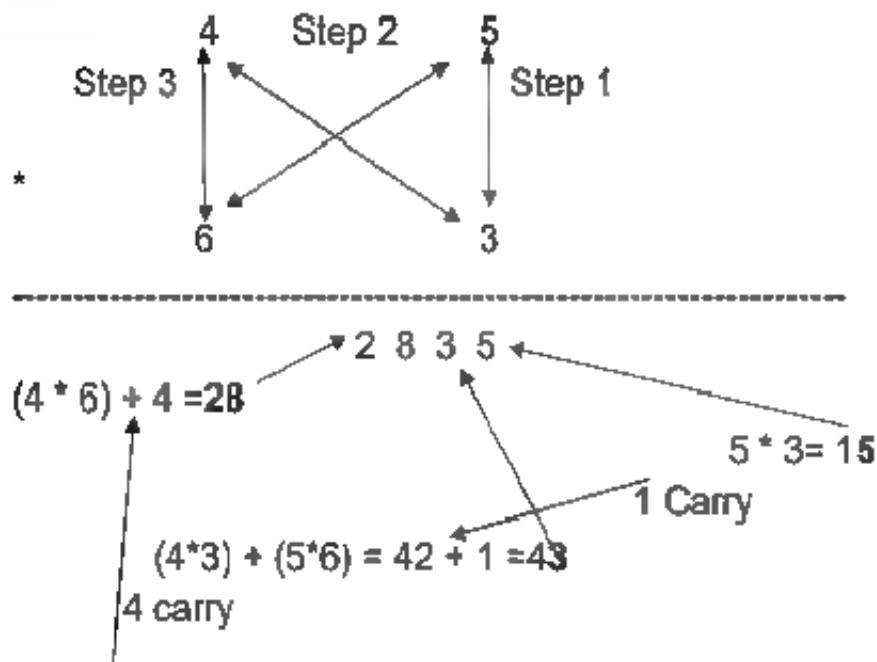
| Step   | Description                           | Calculation   | Answer |
|--------|---------------------------------------|---|--------|
| Step 1 | Multiply rightmost 2 digits           | $8 * 8 = 64$<br>[write 4 and <b>carry 6</b> ]   | 4      |
| Step 2 | Multiply diagonal digits and add them | $(7 * 8) + (8 * 3) = 56 + 24$   64<br>$= 80$<br>$80 + \text{Previous carry}$<br>$80 + 6 = 86$<br>[ write 6 and <b>carry 8</b> ] |        |
| Step 3 | Multiply left most 2 digit            | $7 * 3 = 21$<br>$21 + \text{Previous carry}$<br>$21 + 8 = 29$   | 2964   |

**Answer is 2964***Now will solve next 2 problems orally in single line*

**Problem 4:**

$$45 \times 63$$

**Solution:**

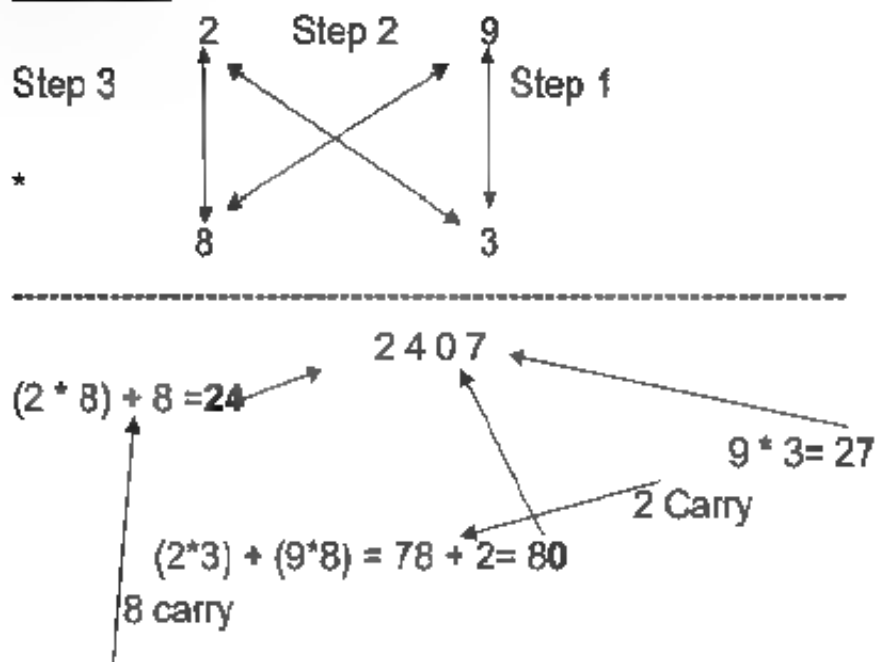


**Answer is 2835**

**Problem 5:**

$$29 \times 83$$

**Solution:**



**Answer is 2407**

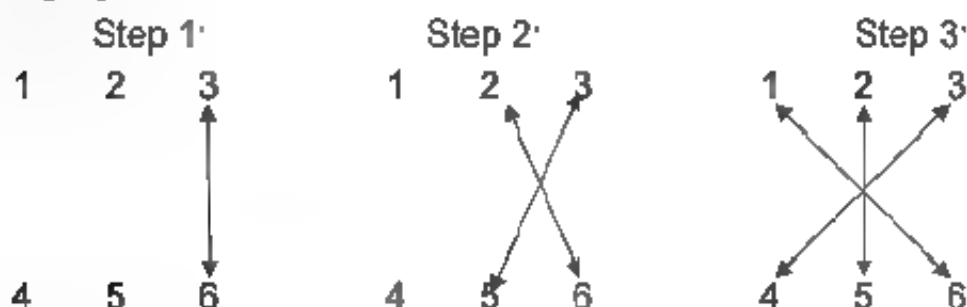
## Type 2: 3 digit number \* 3 digit number

## Problem 1:

$$123 \quad * \quad 456$$

Solution:

[Right-Most 2 digit] [Diagonally right 2] [Extreme diagonal n middle 2 digits]



| Step   | Description   | Calculation  | Answer |
|--------|---|--|--------|
| Step 1 | Multiply Right-Most 2 digit                               | $3 * 6 = 18$<br>[write 8 and <b>carry 1</b> ]  | 8      |
| Step 2 | Multiply right side diagonal digits and add them          | $(2 * 6) + (3 * 5) = 12 + 15$<br>$= 27$<br>$27 + \text{Previous carry}$<br>$27 + 1 = 28$<br>[ write 8 and <b>carry 2</b> ] | 88     |
| Step 3 | Multiply extreme diagonal 2 digit , middle digits and add | $(1*6)+(3*4) +(2*5) = 28$<br>$28 + \text{Previous carry}$<br>$28 + 2 = 30$   | 088    |

|        | them   | [write 0 and carry 3]  |
|--------|--|--|
| Step 4 | Multiply left side diagonal digits and add them. | $(1 \times 5) + (2 \times 4) = 13$<br>$13 + \text{Previous carry}$<br>$13 + 3 = 16$<br>[write 6 and carry 1] |
| Step 5 | Multiply Left Most 2 digits                      | $1 \times 4 = 4$<br>$4 + \text{Previous carry}$<br>$4 + 1 = 5$   |

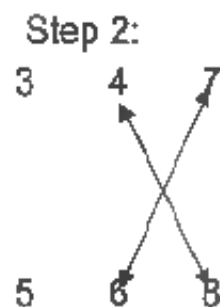
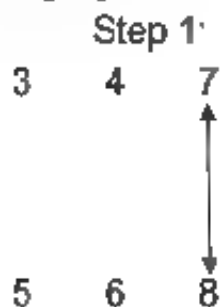
**Answer is 56088**

**Problem 2:**

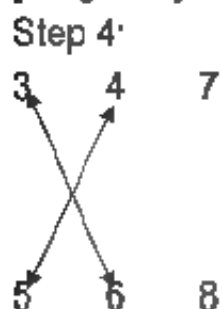
$$347 \times 568$$

**Solution:**

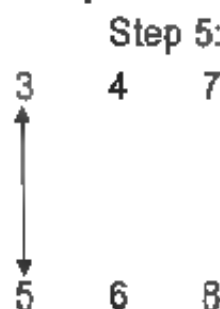
[Right-Most 2 digit]    [Diagonally right 2]    [Extreme diagonal n middle 2 digits]



[Diagonally left 2]



[Left-Most 2 digit]





| Step   | Description  | Calculation   | Answer |
|--------|--|---|--------|
| Step 1 | Multiply Right-Most 2 digit                                    | $7 \times 8 = 56$<br>[write 6 and carry 5]  | 6      |
| Step 2 | Multiply right side diagonal digits and add them               | $(4 \times 8) + (7 \times 6) = 32 + 42$<br>$= 74$<br>$74 + \text{Previous carry}$<br>$74 + 5 = 79$<br>[ write 9 and carry 7]                    | 96     |
| Step 3 | Multiply extreme diagonal 2 digit , middle digits and add them | $(3 \times 8) + (7 \times 5) + (4 \times 6)$<br>$= 24 + 35 + 24 = 83$<br>$83 + \text{Previous carry}$<br>$83 + 7 = 90$<br>[write 0 and carry 9] | 096    |
| Step 4 | Multiply left side diagonal digits and add them                | $(3 \times 6) + (4 \times 5) = 38$<br>$38 + \text{Previous carry}$<br>$38 + 9 = 47$<br>[write 7 and carry 4]                                    | 7096   |
| Step 5 | Multiply Left Most 2 digits                                    | $3 \times 5 = 15$<br>$15 + \text{Previous carry}$<br>$15 + 4 = 19$  | 197096 |

**Answer is 197096**

**Problem 3:**

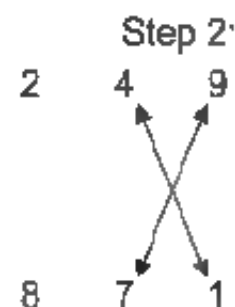
$249 \times 871$

**Solution:**

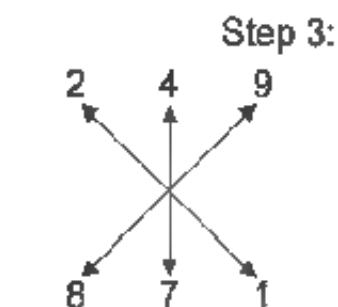
[Right-Most 2 digit]



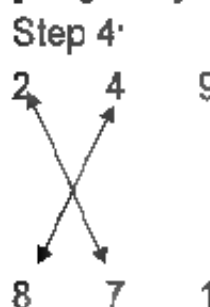
[Diagonally right 2]



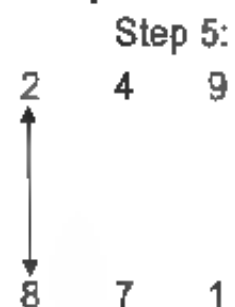
[Extreme diagonal n middle 2



[Diagonally left 2]



[Left-Most 2 digit]



| Step   | Description  | Calculation   | Answer |
|--------|--|---|--------|
| Step 1 | Multiply Right-Most 2 digit                                    | $9 \times 1 = 9$<br>[write 9]   | 9      |
| Step 2 | Multiply right side diagonal digits and add them               | $(4 \times 1) + (9 \times 7) = 4 + 63 = 67$<br>$67 + \text{Previous carry}$<br>$67 + 0 = 67$<br>[ write 7 and carry 6]    | 79     |
| Step 3 | Multiply extreme diagonal 2 digit , middle digits and add them | $(2 \times 1) + (9 \times 8) + (4 \times 7)$<br>$= 2 + 72 + 28 = 102$<br>$102 + \text{Previous carry}$<br>$102 + 6 = 108$ | 879    |

|        |   |   |        |
|--------|---|---|--------|
|        |   | [write 8 and carry 10]  |        |
| Step 4 | Multiply left side diagonal digits and add them | $(2 * 7) + (4 * 8) = 46$<br>$46 + \text{Previous carry}$<br>$46 + 10 = 56$<br>[write 6 and carry 5] | 6879   |
| Step 5 | Multiply Left Most 2 digits                     | $2 * 8 = 16$<br>$16 + \text{Previous carry}$<br>$16 + 5 = 21$                                       | 216879 |

**Answer is 216879**

Now will solve next 2 problem orally in single line

**Problem 4:**

$$145 * 963$$

**Solution:**

$$\begin{array}{r}
 1 \quad \quad \quad 4 \quad \quad \quad 5 \\
 * \quad 9 \quad \quad \quad 6 \quad \quad \quad 3
 \end{array}$$

---

$(1*9)=9 + 4 = 13$  → 1 3 9 6 3 5  
 $(1*6) + (4*9) = 42 + 7 = 49$  →  
 Carry 4  
 $[(1 * 3) + (5 * 9) + (4 * 6) = 72 \text{ and } 72 + 4] = 76$  →  
 Carry 7  
 $(4 * 3) + (5 * 6) = 42 \text{ and } 42 + 1 = 43$  →  
 Carry 4  
 $15 [5 * 3]$  →  
 Carry 1

**Answer is 139635**

**Problem 5:**

$$293 \times 829$$

**Solution:**

|   |   |    |
|---|---|----|
| 2 | 9 | 3  |
| * | 8 | 29 |

---

$(2 \times 8) = 16 + 8 = 24$  → 2 4 2 8 9 7

$(2 \times 2) + (9 \times 8) = 76 + 6 = 82$   
Carry 8

$[(2 \times 9) + (3 \times 8) + (9 \times 2) = 60 \text{ and } 60 + 8] = 68$   
Carry 6

$(9 \times 9) + (2 \times 3) = 87 \text{ and } 87 + 1 = 89$   
Carry 8

27  $[3 \times 9]$   
Carry 2

**Answer is 242897**

**Type 3:** 2 digit number \* 3 digit number

**Problem 1:**

$$45 * 963$$

**Solution:** Just add extra 0 to 2 digit number and follow same procedure

|   |   |   |
|---|---|---|
| 0 | 4 | 5 |
| * | 6 | 3 |

---

$(0*9)=0 + 4 = 4$

$(0*6) + (4*9) = 36 + 7 = 43$   
Carry 4

$[(0 * 3) + (5 * 9) + (4 * 6) = 69 \text{ and } 69 + 4] = 73$   
Carry 7

4 3 3 3 5

$15 [5 * 3]$   
Carry 1

$(4 * 3) + (5 * 6) = 42 \text{ and } 42 + 1 = 43$   
Carry 4

**Answer is 43335**

**Type 4:** 3 digit number \* 2 digit number

**Problem 1:**

$$145 * 63$$

**Solution:** Just add extra 0 to 2 digit number and follow same procedure

|   |   |   |   |
|---|---|---|---|
| 1 | 4 | 5 |   |
| * | 0 | 6 | 3 |

---

(1\*0)=0 = 0 →

(1\*6) + (4\*0) = 6 + 3 = 9 →

[(1 \* 3)+(5 \* 0)+(4 \* 6)=27 and 27 + 4] =31  
Carry 3

0 9 1 3 5

→ 15 [5 \* 3]  
Carry 1

→ (4 \* 3) + (5 \* 6) = 42 and 42 + 1 = 43  
Carry 4

**Answer is 9135**

**Mental Calculation Tricks****Addition**

Conventional Method

**Problem 1**

$$76 + 65$$

So you will write this down

**Step1**

You will add unit digit number 6 and 5 So sum is 11 You will write 1 and carry 1.

**Step2**

You will add 10th place digits 7 and 6 and will add 1 carry that is  $13 + 1 = 14$   
So answer is 141

With conventional method, you will need at least 15-20 seconds to solve addition including writing numbers and add

Let us try to do this orally and we can definitely do this orally in less than 10 seconds and with good amount of practice you can do in less than 5 seconds

**.Method 1**

Consider unit digit as 0 and add numbers

Add unit digits sum to this total.

**Problem 1:**

$$76 + 65$$

**Solution :**

Let us ignore unit digit and consider them as 0.

70 and 60

So anyone can add these 2 numbers orally easily

$$70 + 60 = 130$$

Now add unit place digits 6 and 5 that is 11

Now add 11 to 130

That will become 141

**Answer is 141**

**Problem 2:**

$$93 + 56$$

**Solution :**

Let us assume unit place digit as 0

So 90 and 50 sum is 140

Now add unit place digits 3 and 6 that is 9

Now add 9 to 140

**Answer is 149**

**Problem 3**

$$123 + 348$$

**Solution :**

Let us assume unit place digit as 0 So adding 120 and 340 is easy  
(12+34=46) so 460

Unit digit place 3 and 8 that is 11

Now add 11 to 460 that is 471

**Answer is 471**

**Subtraction:****Method 1**

Let us try to solve below few examples orally before we look at trick for subtraction

Note down time required to solve Problem 1 to Problem 5 orally

**Problem1:**

$$62 + 900$$

**Problem2:**

$$720 + 82$$

**Problem3:**

$$750 + 65$$

**Problem4:**

$$98 + 510$$

**Problem5:**

$$66 + 170$$



Note down time required to solve Problem 6 to Problem 10 orally

**Problem6:**

$$510 - 98$$

**Problem7:**

$$720 - 82$$

**Problem8:**

$$750 - 65$$

**Problem9:**

$$170 - 66$$

**Problem10**

$$900 - 62$$

We can guarantee that for all people time required to solve Problem6 to Problem10 is larger than solving Problem1 to Problem5.

Why this happen?

Our brain is really comfortable in adding number. Like everyone wants money and want to be happy No one likes to give money to other and being sad in loss of money.

Therefore addition becomes easy and happy while subtraction becomes hard and sad

Let us try to do Subtraction in some other way than usual method

**Problem 6:**

$$510 - 98$$

**Solution :**

Now don't try to subtract 98 from 510

Think how much I need to add to 98 so that it will become 510

So in 98

If we add 100 then 198

If we add 200 then 298

Like this if we add 400 it will become 498

As we reached 498 which is very close to 510

So we need to add 12 more to get 510

So  $400 + 12 = 412$

**Answer for  $510 - 98 = 412$**

**Problem 7:**

$720 - 82$

**Solution :**

Now think how much to be added in 82 to get 720

If we add 100 then 182

If we add 200 then 282

Like this if we can add 600 it will become 682

Now how much more we need to add in 682 to get 720

If you are feeling getting 682 to 720 difficult then go for nearest century that is 700

So if we add 18 in 682 that becomes 700 and now only 20 remaining ( $720 - 700 = 20$ )

So we added  $600 + 18 + 20 = 638$

**Answer is 638**

**Method 2**

Doing addition is very simple for anyone. When it comes to Subtraction, many of us face bit challenging.

It is because the way we handle carry in additions is much simpler than we use in Subtraction.

Consider example below and try to do Subtraction in different and simple way

**Example 1 (Without any carry)**

$9766 - 2354$

$$\begin{array}{r}
 9766 \\
 -2354 \\
 \hline
 7412
 \end{array}$$

**Step 1**

Instead of doing  $6 - 4$

Think some number  $+ 4 = 6$

292

What is that some number? 2

As  $2 + 4 = 6$

**Step 2**

Some number  $+ 5 = 6$

Some number is 1

**Step 3**

Some number  $+ 3 = 7$

Some number is 4

**Step 4**

Some number  $+ 2 = 9$

Some number is 7

Answer is 7412

**Example 2 (With carry)**

2000 - 765

2 0 0 0

- 7 6 5

-----

1 2 3 5

1 1 1

**Step 1**

Instead of doing  $0 - 5$

Think some number  $+ 5 = 0$

Its not possible as 5 is bigger than 0

So we need to consider 10 instead of 0

Some number  $+ 5 = 10$

Some number is 5

As we have written 0 , 1 is carry which we will write at bottom and not at top

**Step 2**

1 (carry) + Some number  $+ 6 = 0$

Again as 6 is bigger than 0, we need to consider 0 as 10

$$1 + \text{Some number} + 6 = 10$$

Some number is 3

Again we will write carry 1 at bottom

**Step 3**

$$1 + \text{Some number} + 7 = 10$$

Some number is 2

Write carry 1 at bottom

**Step 4**

$$1 + \text{Some number} = 2$$

Some number is 1

**\*\*Answer is 1235\*\***

**Example 3 (With carry)**

$$2360 - 980$$

$$2360$$

$$-980$$

-----

$$1375$$

$$111$$

**Step 1**

$$293$$

$$\text{Some number} + 5 = 10$$

Some number is 5

1 is carry which we will write at bottom

**Step 2**

$$1 (\text{carry}) + \text{Some number} + 8 = 6$$

Again as 6 is bigger than 8, we need to consider 6 as 16

$$1 (\text{Carry}) + \text{Some number} + 8 = 16$$

Some number is 7

Again we will write carry 1 at bottom

**Step 3**

$$1 + \text{Some number} + 9 = 13$$

Some number is 3

Write carry 1 at bottom

**Step 4**

$1 + \text{Some number} = 2$

Some number is 1

**Answer is 1375**

**Multiplication**

We are going to look at some shortcut tricks for multiplication. This technique can be applied on all multiplication.

However you may feel/found it useful for some cases and may not be useful for other cases. So please go through this method and if you can apply for few places that will save lot of time.

This technique should help most of the places where we have 1 multiplication number as Factor of 100, 10, 1000 etc.

Try to solve below 5 problems orally and note down time for solving these problems:

**Problem 1:**

$$52 * 25$$

**Problem 2:**

$$50 * 65$$

**Problem 3:**

$$20 * 65$$

**Problem 4:**

$$25 * 92$$

**Problem 5:**

$$83 * 500$$

Let us try to solve these example with shortcut trick

If we need to find  $a * b$

We can divide  $a$  by  $n$  and multiply  $b$  by  $n$ .

So basically we are multiplying by  $n/n$  that is 1 on  $y$   
( $n$  can be any natural number 2, 3, 4 ... etc)

You may feel what's point in doing this extra calculation. But you will see these small things make a huge difference and good time saver.

**Problem 1:**

$$52 * 25$$

**Solution :**

As you can see 25 is factor of 100 and 52 is completely divisible by 4

So we will divide 52 by 4 and multiply 25 by 4

$$(52 / 4) * (25 * 4) = 13 * 100$$

This is very easy to solve

**Answer is 1300**

**Problem 2:**

$$50 * 65$$

**Solution :**

As you can see we have 50 as factor of 100.

So we will divide 65 by 2 and multiply 50 by 2

$$(50 * 2) * (65/2)$$

$$= 100 * 32.5$$

**Answer is 3250**

**Problem 3:**

$$20 * 65$$

**Solution :**

As you can see 20 is factor of 100 (we will prefer to multiply by 5) and 65 is completely divisible by 5.

$$(20 * 5) * (65 / 5)$$

$$278$$

$$= 100 * 13$$

**Answer is 1300**

**Squares:**

Calculating square is very much important while solving Quantitative Aptitude problems

We are going to see small tricks to calculate square orally

This is very basic tricks and in few days we will look at some advanced trick where we can calculate square of 2 or 3 digit number orally in less than 10 seconds.

**Trick 1: Number ending with 5 only**

**Assume you have to find square of 65.**

Everyone knows square of 5 is 25

Write 25 to right most

As we found square of 5, remaining digit is 6

We need to add 1 in this remaining digit and multiply both

$$6 * (6 + 1)$$

$$= 6 * 7 = 42$$

So our answer is 4225

**Problem 1:**

$$75^2$$

**Solution :**

As unit digit is 5, find square of 5 that is 25 Write 25 at rightmost

Now pending digit is 7

We need to add 1 in 7 and multiply them

$$7 * (7 + 1) = 7 * 8$$

$$= 56$$

**Answer is 5625**

**Problem 2:**

$$85^2$$

**Solution :**

As unit place digit is 5, find square of 5 first that is 25 Write 25 at rightmost

Now pending digit is 8 So add 1 in 8 and multiply both of them

$$8 * (8 + 1) = 8 * 9$$

$$= 72$$

**Answer is 7225**

**Problem 3:**

$$205^2$$

**Solution :**

As unit digit number is 5 , find square of 5 that is 25. Write 25 at right most. Now pending digit is 20. So we need to add 1 in 20 and multiply both of them

$$20 * (20 + 1) = 20 * 21$$

$$= 420$$

**Answer is 42025**

**Trick 2: When you know  $n^2$  and you need to find  $(n+1)^2$** 

**Assume you have to find square of 61**

Calculating square of 60 is very simple than calculating square of 61.

Let us calculate square of 60

$$60^2 = 3600$$

Now add same number and number whose square to be find out

$$3600 + 60 + 61 = 3600 + 121$$

$$= 3721$$

**Answer is 3721**

**Problem 1:**

$$71^2$$

**Solution :**

Calculating square of 70 is very simple than calculating square of 71.

Let us calculate square of 70

$$70^2 = 4900$$

Now add same number and number whose square to be find out

$$4900 + 70 + 71 = 4900 + 141 = 5041$$

**Answer is 5041**

**Problem 3 :**

$$106^2$$

**Solution :**

We know that finding square for 5 is very simple than 106 ( using Trick for number ending with 5)



$$105^2 = 10 \times 11 \text{ followed by } 25 \\ = 11025$$

Now add 100 and 101 in it

$$= 11025 + 105 + 106$$

$$= 11025 + 211$$

$$= 11236$$

**Answer is 11236**

You can use this trick for majority number ending with 1 or 6 as finding square of number ending with 0 and 5 is very easy

### **Trick 3: Any number square**

You may feel this as simplified version of  $a^2 + 2ab + b^2$

But I guarantee once you understand this trick you no longer need to remember formula or probably you will never forget it.

#### **Pre-requisite:**

1 to 9 squares should be known.

It is nice to have to know till 30.

### **Problem 1**

Assume you have to find square of 69

#### **Solution :**

Split into 2 parts 6 and 9.

#### **Step 1:**

Start from right side

Take 9 and  $9^2 = 81$

So write 1 to rightmost and carry is 8.

#### **Step 2:**

Multiply both digits and double.

$$(6 \times 9) \times 2 = 54 \times 2 = 108$$

Step1 Carry was 8 So  $108 + 8 = 116$

Write 6 and carry 11

61

#### **Step 3:**

Take leftmost digit 6

$$6^2 = 36$$

Step2 carry was 11

$$36 + 11 = 47$$

**Answer is 4761**

**Problem 2:**

$$82^2$$

**Solution :**

Split into 2 parts 8 and 2

**Step 1:**

Start from right side

$$280$$

Take 2 and  $2^2 = 4$

So write 4 to rightmost.

**Step 2:**

Multiply both digits and double.

$$(8 * 2) * 2 = 16 * 2 = 32$$

So write 2 and carry is 3

$$24$$

**Step 3:**

Take leftmost digit 8

$$8^2 = 64$$

Step2 carry was 3

$$64 + 3 = 67$$

**Answer is 6724**

**Problem 3:**

$$184^2$$

**Solution :**

Here let us divide number into 2parts 18 and 4

**Step 1:**

Start from right side

Take 4 and  $4^2 = 16$

So write 6 to rightmost and carry is 1

**Step 2:**

Multiply both digits and double.

$$(18 \times 4) \times 2 = 72 \times 2 = 144$$

Step1 carry was 1

$$144 + 1 = 145$$

So write 5 and carry is 14

56

**Step 3:**

Take left digit 18

$$18^2 = 324$$

Step2 carry was 14

$$324 + 14 = 338$$

**Answer is 33856**

**Note:** Right side part should be a ways rightmost 1 digit only and other left part is 2nd part.

Like for 105 ·

10 and 5 are 2 parts

For 321

32 and 1 are 2 parts

If you feel left part is bigger you can follow same method to find square of 1st part.

You can use this trick for any square and once you practice you can really find any 2 or 3 digit number in less than 10 seconds orally

**Cross multiplication**

Cross Multiplication is very common thing which we across most of Aptitude topics.

Whether its profit loss, unit price of any item or time and distance calculation etc.

Most of you would be already knowing this trick still I wanted to share so that whoever not aware will get different viewpoint for calculation orally.

**Problem 1:**

If 10 pen cost is 500 Rupees what is cost of 25 Pens

10-----500

25-----?

**Solution :**

In this case most of people do below calculation:

$$500 * 25 / 10 = 12500/10 = 1250$$

(This calculation will be very challenging to do orally)

Now lets look at this with different viewpoint and do it orally

283

With what number we should multiply 10 so that we get 25

$$10 * 2.5 = 25$$

So in order to find answer in place of ? we will multiply 500 by 2.5

$$500 * 2.5 = 1250$$

**Answer is 1250**

**Problem 2 :**

If Seller earns 25% profit on some item What will be Cost price for item if selling price is 500Rs?

100-----125

? ----- 500

**Solution :**

With what number we should multiply 125 to get 500

$$125 * 4 = 500$$

So in order to find answer in place of ? we need to multiply 100 by 4

$$100 * 4 = 400$$

**Answer is Cost price is 400**

### **Mean, Median and Mode Concept**

All these 3 terms are really simple However they all are based on getting central idea of anything so it becomes always confusing to solve for most of the people

Let us try to understand these terms in very simple way so at any point of time you will never get confused in these 3

terms

**MeAn - Mean Average** ( take normal average)

**MeDian- Mid Point** ( Arrange all numbers in ascending that is smallest to highest and take middle number. If values are even then take average of 2 middle number in series)

**MOde- More** time ( number most of times)

**Problem 1:**

Find Mean, Median and mode for

2, 2, 4, 4, 4, 3, 6, 7

**Solution :**

Mean -  $(2 + 2 + 4 + 4 + 4 + 3 + 6 + 7) / 8$

Mean =  $32 / 8 = 4$

Median : Arrange in ascending order

2, 2, 3, 4, 4, 4, 6, 7

Now as number of values even, we need to take average of middle 2 numbers

Median =  $(4 + 4) / 2 = 4$

Mode -

2 occurred 2 times

3, 6, 7 occurred 1 time

4 is occurred maximum times in series that is 3 times.

**Mode is 4**

Now just remember as this and any time you write these words try to put in way

When you write MeAn where M and A is capital you will be able to memorize A is for Average

When you write MeDian M and D as capital so you will recollect MidPoint

When you write MOde write MO capital so you will remember More

**Quadratic Equation****Trick 1**

Quadratic equation is very important while solving Aptitude problems. Many students face challenge to solve Quadratic Equations. In order to get expertise in solving quadratic equation

We will do following activity

Splitting middle term is the major part of solving quadratic equation which we are going to look at with very simple numbers calculation (forget about  $x$  and  $x^2$  for now)

If you practice below kind of 20-30 examples and you know tables 1 to 30 it would be very easy to solve any quadratic equation.

**Problem 1**

Sum of 2 numbers 18 and multiplication is 80

**Solution****Step 1**

Now try to identify factors of 80.

1, 2, 4, 8, 10, 16, 20, 40, 80

**Step2**

Now identify 2 factors whose sum is 18 and Multiplication is 80

8 and 10 sum is 18 and multiplication is 80

Just to correlate this

$$x^2 + 18x + 80 = 0$$

$$x^2 + 10x + 8x + 80 = 0$$

$$x(x + 10) + 8(x + 10) = 0$$

$$(x + 10)(x + 8) = 0$$

$$\text{So } x = -10 \text{ or } x = -8$$

**Problem 2**

Sum of 2 numbers 19 and multiplication is 60

**Solution :****Step 1**

Now try to identify factors of 60.

1, 2, 4, 6, 10, 12, 30, 60

**Step2**

Now identify 2 factors whose sum is 19 and multiplication is 60

15 and 4 sum is 19 and multiplication is 60

Just to correlate this

$$x^2 + 19x + 60 = 0$$

$$x^2 + 15x + 4x + 60 = 0$$

$$x(x + 15) + 4(x + 15) = 0$$

$$(x + 4)(x + 15) = 0$$

$$x = -4 \text{ or } x = -15$$

Practice below problems where sum of 2 numbers and multiplication given

- 1) Sum 14 and Multiplication 40
- 2) Sum 16 and Multiplication 63
- 3) Sum 22 and Multiplication 120
- 4) Sum 23 and Multiplication 90
- 5) Sum 16 and Multiplication 63
- 6) Sum 18 and Multiplication 77
- 7) Sum 20 and Multiplication 64
- 8) Sum 15 and Multiplication 56

**Trick 2****Problem 1**

Difference of 2 numbers is 2 and multiplication is 80

**Solution :**

**Step 1**

Now try to identify factors of 80.

1, 2, 4, 8, 10, 16, 20, 40, 80

**Step 2**

Now identify 2 factors whose difference is 2 and Multiplication is 80

10 - 8 = 2 and multiplication is 80

Just to correlate this

$$x^2 + 2x - 80 = 0$$

$$x^2 + 10x - 8x - 80 = 0$$

$$x(x + 10) - 8(x + 10) = 0$$

$$(x + 10)(x - 8) = 0$$

$$x = -10 \text{ or } x = 8$$

**Problem 2**

Difference of 2 numbers is 11 and multiplication is 60

**Solution :**

**Step 1**

Now try to identify factors of 60.

1, 2, 4, 6, 10, 12, 15, 20, 30, 60

**Step 2**

Now identify 2 factors whose difference is 11 and multiplication is 60

$15 - 4 = 11$  and multiplication is 60

Just to correlate this

$$x^2 + 11x - 60 = 0$$

$$x^2 + 15x - 4x - 60 = 0$$

$$x(x + 15) - 4(x + 15) = 0$$

$$(x - 4)(x + 15) = 0$$

$$x = 4 \text{ or } x = -15$$

Practice below problems where difference of 2 numbers and multiplication given

- 1) Difference 14 and Multiplication 72
- 2) Difference 16 and Multiplication 80
- 3) Difference 22 and Multiplication 75
- 4) Difference 23 and Multiplication 210
- 5) Difference 16 and Multiplication 36
- 6) Difference 18 and Multiplication 63
- 7) Difference 20 and Multiplication 69
- 8) Difference 15 and Multiplication 100



**Trick 3****Problem 1**

$$x^2 + 7x + 10$$

**Solution :****Step 1**

Look at all + or - signs Very Important.

First term =  $x^2$ Second Term =  $(+7x)$ Thrd term =  $(+10)$ Split Second Term  $(+7x)$  in such way that multiplication of split values would be product of First Term  $(+x^2)$  and Third

$$\text{Term}(+10) = x^2 * 10 = 10x^2$$

**Step 2**We can say if we split Second Term  $(7x)$  into  $2x$  and  $5x$  then  $2x + 5x = 7x$ And  $2x * 5x = 10x^2$  ( As per expectation)

$$x^2 + 7x + 10 = 0$$

$$x^2 + 2x + 5x + 10 = 0$$

$$x(x + 2) + 5(x + 2) = 0$$

$$(x + 5)(x + 2) = 0 \text{ as } (x + 2) \text{ is common}$$

$$x + 5 = 0 \text{ or } x + 2 = 0$$

**Answer is  $x = -5$  or  $x = -2$** **Problem 2**

$$x^2 - 9x - 22$$

**Solution :****Step 1**

Look at all + or - signs Very Important.

First term =  $x^2$ 

288

Second Term =  $(-9x)$ Thrd term =  $(-22)$ Split Second Term  $(-9x)$  in such way that multiplication of split values would be product of First Term  $(+x^2)$  and Third

$$\text{Term}(-22) = x^2 * (-22) = -22x^2$$

**Step 2**

We can say if we split Second Term(-9x) into (-11x) and (+2x) then  $-11x + 2x = -9x$

And  $(-11x) \times (2x) = -22x^2$  ( As per expectation)

$$x^2 - 9x - 22 = 0$$

$$x^2 - 11x + 2x - 22 = 0$$

$$x(x - 11) + 2(x - 11) = 0$$

$$(x + 2)(x - 11) = 0 \text{ as } (x - 11) \text{ is common}$$

$$x + 2 = 0 \text{ or } x - 11 = 0$$

**Answer is  $x = -2$  or  $x = 11$**

### **Type of Numbers**

Every work we do in our day to day life, basic concepts are very much important than any advanced concepts.

Any profession you consider, if you forget / ignore basics then it will be challenging for you to understand or tackle any real time problem

Like if you switch on fan and fan does n't starts, most basic thing you should check is that electricity is there or not. Similarly many of us does graduation, prepares for competitive exam, become engineers or some other professional but we forget Basics of numbers

#### **Natural Numbers**

Anything which we used to count (1 chocolate, 10 chocolates etc ) are natural Numbers. 0 is not Natural number as we can't count it

Eg 1,2,3. . . .

#### **Whole Numbers**

All Natural numbers and considering 0 as well it makes Whole numbers.

Eg 0,1,2,3 ..

#### **Integers**

All positive and negative numbers including 0 are called as Integers.

Eg . . -3, -2, -1, 0, 1, 2, 3 . . .

#### **Even Numbers**

Any number which is divisible by 2 is even number. In simple words, any number ending with 0 or 2 or 4 or 6 or 8 is even number

Eg 2, 4, 6, 18, 200 etc...

**Odd Numbers**

Any number which is not divisible by 2 is odd number. In simple words, any number ending with 1 or 3 or 5 or 7 or 9 is Odd number

Eg 1, 43, 65, 77, 209 etc .

**Prime Numbers**

Any number which is having exact 2 factors is called as prime number. 1 is not prime number as it has only 1 factor which is 1.

Eg 2, 3, 5, 7, 17 etc

There are 25 prime numbers from 1 to 100

**Composite Numbers**

Any number having more than 2 factors is called Composite number

Eg 6 as it has factors 1, 2, 3, 6

20 as it has factors 1, 2, 4, 5, 10, 20

**CoPrime numbers**

Any 2 or more numbers whose HCF is 1 are Co-prime numbers.

Eg 2 and 5, 10 and 7

**Rational Numbers**

Any number which we can express as  $x/y$  where  $x$  and  $y$  are Integers and  $y$  not equal to 0

Eg  $1/2$ ,  $2/3$

**Irrational Numbers**

Any number which we can not be expressed as  $x/y$  where  $x$  and  $y$  are Integers and  $y$  not equal to 0

Eg Square root 2,  $\pi$  etc.

**Tables Calculation Tricks**

When we solve any of mathematical problems, doing calculations is most common and important thing

Calculations would be simple if we are capable of memorising tables 1 to 30

Most of people feel comfortable memorising table of 5, 10, 15, 20 or 25

Specifically numbers which are ending with 5 and 0 as our brain becomes very comfortable in these calculations

However if we try to memorize table of 19, 29, 21 we face lot of challenges  
Here is very simple and basic trick to memorise these tables

**Example 1: Table of 21**

$$21 * 1 = (20 * 1) + (1 * 1)$$

In mind you can do  $20 * 1$  is 20 and  $1 * 1$  is 1 so it becomes  $20 + 1 = 21$

$$21 * 2 = (20 * 2) + (1 * 2)$$

In mind you can do  $20 * 2$  is 40 and  $1 * 2$  is 2 so it becomes  $40 + 2 = 42$

..

$$21 * 8 = (20 * 8) + (1 * 8)$$

In mind you can do  $20 * 8$  is 160 and  $1 * 8$  is 8 so it becomes  $160 + 8 = 168$

$$21 * 9 = (20 * 9) + (1 * 9)$$

In mind you can do  $20 * 9$  is 180 and  $1 * 9$  is 9 so it becomes  $180 + 9 = 189$

$$21 * 10 = 210$$

**Example 2: Table of 19**

$$19 * 1 = (20 * 1) - (1 * 1)$$

In mind you can do  $20 * 1$  is 20 and  $1 * 1$  is 1 so it becomes  $20 - 1 = 19$

$$19 * 2 = (20 * 2) - (1 * 2)$$

In mind you can do  $20 * 2$  is 40 and  $1 * 2$  is 2 so it becomes  $40 - 2 = 38$

..

$$19 * 8 = (20 * 8) - (1 * 8)$$

In mind you can do  $20 * 8$  is 160 and  $1 * 8$  is 8 so it becomes  $160 - 8 = 152$

$$19 * 9 = (20 * 9) - (1 * 9)$$

In mind you can do  $20 * 9$  is 180 and  $1 * 9$  is 9 so it becomes  $180 - 9 = 171$

$$19 * 10 = 190$$

**This technique will help you to memorize all tables by referring to tables for numbers ending with 0, 5**

**Direct and Inverse Proportion****Concept of Direct Proportion**

Let us try to understand the concept of direct proportion in very simple words

If you can purchase 5 maggi packets in 100 Rs then how much money you will need to purchase 10 maggi packets ?

Option A: More money than 100 Rs

Option B: Less money than 100 Rs

Obviously, we know option A is correct, as we are buying more number of Maggie packets than earlier, we will need more than 100rs.

→ If we increase the quantity of one item, it also increases quantity of other item, we can say item 1 and item 2 are in direct proportion

**Few examples of Direct proportion:**

1. Number of items purchased and total price
2. New home area and price of home
3. Petrol filled in car and distance travel capacity

**Concept of Inverse Proportion**

Let us try to understand the concept of inverse proportion in very simple words

If you start walking from your home to school, you reach school in 20 minutes. If you travel by car or bike then how much time you will need to reach school?

Option A: More than 20 minutes

Option B: Less than 20 minutes

Obviously, we know option B is correct, as we know bike or car will definitely go with faster speed than our walking speed.

→ If we increase the quantity of 1st item, which decreases quantity of 2<sup>nd</sup> item, it means there is inverse proportion

**Few examples of direct proportion:**

1. Speed of car and time required to travel
2. Number of people doing work and time required to complete work

**Direct Proportion: Constant**

Let us assume 2 different quantities

- A. Number of Maggie packets purchased
- B. Amount paid

When 2 quantities a and b are in direct proportion, value of a/b is always remains constant. Find the value at place of ? (a and b are in direct proportion)

|   |     |    |      |    |
|---|-----|----|------|----|
| a | 5   | 20 | ?    | 65 |
| b | 100 | ?  | 2000 | ?  |

**Solution:**

1) we know a and b are in direct proportion. So for any pair of a and b a/b should be constant.

$$\frac{a}{b} = \frac{5}{100} = \frac{1}{20}$$

**For every pair we should get a/b=1/20**

1) If a = 20,

$$\frac{1}{20} = \frac{20}{b}$$

$$1 \times b = 20 \times 20$$

$b = 400$

2) If b = 2000,

$$\frac{1}{20} = \frac{a}{2000}$$

$$1 \times 2000 = 20 \times a$$

$$a = 100$$

3] If  $a = 65$ ,

$$\frac{1}{20} = \frac{65}{b}$$

$$1 \times b = 65 \times 20$$

$$b = 1300$$

### Direct and Inverse Proportion: Direct Proportion Check

Check whether a and b vary directly or not.

**Problem 1 :**

|   |    |    |    |    |
|---|----|----|----|----|
| a | 4  | 7  | 21 | 28 |
| b | 12 | 21 | 63 | 84 |

### Solution:

we know that when ever a and b are in direct proportion for any value of a/b is always constant.

Here verifying each a/b with each other by cross multiplication will be time consuming So convert each a/b into simplest form and compare

$$\frac{4}{12} = \frac{1}{3}, \quad \frac{7}{21} = \frac{1}{3}, \quad \frac{21}{63} = \frac{1}{3}, \quad \frac{28}{84} = \frac{1}{3}$$

We see all places we got  $\frac{1}{3}$  on y

So, a and b vary directly

**Problem 2 :**

|   |   |    |    |    |
|---|---|----|----|----|
| a | 2 | 5  | 12 | 21 |
| b | 8 | 20 | 48 | 64 |

**Solution:**

Let us do the simplest form for each a/b

$$\frac{2}{8} = \frac{1}{4}, \quad \frac{5}{20} = \frac{1}{4}, \quad \frac{12}{48} = \frac{1}{4}, \quad \frac{21}{64} = \frac{21}{64}$$

Here we see for last pair value is not same as others 1/4

So, a and b are not in direct proportion

**Direct and Inverse Proportion: Direct Proportion Problems****Problem 1:**

A car travels 432km in 48 liters of petrol. How far car could travel if 20 l ters of petrol is filled ?

**Solution:**

We know car filled in petrol (a) and distance it can cover (b) are in direct proportion

A= Petro filled                      a1 =48      a2 = 20

b= Distance it can travel        b1= 432    b2 = ?

we know a/b always constant

$$\frac{a1}{b1} = \frac{a2}{b2}, \quad \frac{48}{432} = \frac{20}{x}$$

$$48 \times x = 20 \times 432$$

$$x = \frac{20 \times 432}{48}$$



$$x = 20 \times 9$$

$$x = 180$$

Car can travel 180 km in 20 liter petrol

**Problem 2:** Reema types 540 words in half hour How many words she can type in 6 minutes?

**Solution:**

a= Number of words

a1 = 540      a2 = ?

b= Time in minutes

b1 = 30      b2 = 6

$$\frac{a1}{b1} = \frac{a2}{b2}, \quad \frac{540}{30} = \frac{x}{6}$$

$$540 \times 6 = x \times 30$$

$$x = \frac{540 \times 6}{30}$$

$$x = 54 \times 2$$

$$x = 108$$

Reema can type 108 words in 6 minutes.

**Inverse Proportion Constant**

Let us assume 2 different quantities

a Speed of car

b Time required to travel

When 2 quantities a and b are in inverse proportion, value of  $a \times b$  is always remains constant. Find the value at place of ? (a and b are in inverse proportion)

|   |    |     |    |   |
|---|----|-----|----|---|
| a | 30 | 120 | ?  | 5 |
| b | 8  | ?   | 40 | ? |

**Solution:**

we know a and b are in inverse proportion. So for any value of a and b,  $a \times b$  is always constant.

$$a \times b = 30 \times 8 = 240$$

1] If  $a = 120$ ,

$$120 \times b = 240$$

$$b = \frac{240}{120}$$

$b = 2$

2] If  $b = 40$ ,

$$a \times 40 = 240$$

$$a = \frac{240}{40}$$

$a = 6$

3] If  $a = 5$ ,

$$5 \times b = 240$$

$$b = \frac{240}{5}$$

$$b = 48$$

### Inverse Proportion Check

Check whether a and b vary inversely or not.

#### Problem 1 :

|   |   |   |    |    |
|---|---|---|----|----|
| a | 4 | 3 | 12 | 1  |
| b | 6 | 8 | 2  | 24 |

#### Solution:

we know that when ever a and b are in inverse proportion for any value of a and b,  $a \times b$  is always remains constant.

$$4 \times 6 = 24, 3 \times 8 = 24, 12 \times 2 = 24, 1 \times 24 = 24$$

We can see all places we got same value 24

So, a and b vary inversely

#### Problem 2:

|   |   |    |    |    |
|---|---|----|----|----|
| a | 9 | 24 | 12 | 15 |
| b | 8 | 3  | 6  | 4  |

#### Solution:

Let us find value of  $a \times b$  for each pair

$$9 \times 8 = 72, 24 \times 3 = 72, 12 \times 6 = 72, 15 \times 4 = 60$$

Here we can see last pair value is not same 72

So, a and b are not inversely proportion

**Inverse Proportion Problems**

**Problem 1:** If 56 men can complete work in 42 days. How many men can complete in 14 days?

**Solution:** we know that number of men (a) and required number of days to complete work (b) vary inversely.  $a \times b$  is always remains constant

$$a_1 = 56 \quad a_2 = ?$$

$$b_1 = 42 \quad b_2 = 14$$

$$56 \times 42 = a_2 \times 14$$

$$a_2 = \frac{56 \times 42}{14}$$

$$a_2 = 168$$

168 men will be able to complete work in 14 days

**Problem 2:** IN hostel of 50 girls, there is food provision for 40 days. If 30 more girls join hostel, how long will this provision last?

**Solution:** Number of girls (a) and number of days food can last (b) vary inversely. So,  $a \times b$  is constant.

$$a_1 = 50 \quad a_2 = 50 + 30 = 80$$

$$b_1 = 40 \quad b_2 = ?$$

$$50 \times 40 = 80 \times b_2$$

$$b_2 = \frac{50 \times 40}{80}$$

$$b_2 = 25$$

When 30 more girls join hostel, food will be available for 25 days.

**Miscellaneous Problems****Problem 1:**

If 20 men can build wall 56 meters long in 6 days, what length of similar wall can be built by 35 men in 3 days?

**Solution:** Here we have total 3 quantities: men, number of days and length of wall

To make it simple can we say efforts = men \* no. of days. Efforts and length of wall will be directly proportion. Efforts (a) and length of wall (b)

$$a_1 = 20 \times 6 = 120 \quad a_2 = 35 \times 3 = 105$$

$$b_1 = 56 \quad b_2 = ?$$

we know that a/b will always constant

$$\frac{120}{56} = \frac{105}{b_2},$$

$$120 \times b_2 = 105 \times 56$$

$$b_2 = \frac{105 \times 56}{120}$$

$b_2 = 49$

When 35 men work for 3 days 49 m length wall will be built

**Problem 2:** If 15 men, working 9 hours a day, can reap field in 16 days. In how many days will 18 men. reap a field working 8 hours a day?

**Solution:** Here we have total 3 quantities: men, number of hours and days required to reap field.

To make it simple can we say efforts = men \* no. of hours.

Efforts (a) and number of days to reap field (b) are in inversely proportion [more efforts, less days needed or less efforts, more days needed]

$$a_1 = 15 \times 9 = 135 \quad a_2 = 18 \times 8$$

$$b_1 = 16 \quad b_2 = ?$$

a × b will always be constant (as inverse proportion)

$$\frac{120}{56} = \frac{105}{b_2},$$

$$135 \times 16 = 18 \times 8 \times b_2$$

$$b_2 = \frac{135 \times 16}{18 \times 8}$$

$$b_2 = 15$$

When 18 men working 8 hours a day can reap field in 15 days

**Exponent / Power / Index**

When we write any number and write one more number to top write of it, is called as power/ exponent/ index

$2^3$  means 2 to the power 3

Below 2 is the base and top number is power

In simple word if you have to find out value of  $2^3$

Step 1: write 2 number 3 times

Step 2: Give multiplication ( $\times$ ) sign in each pair

$$2 \times 2 \times 2 = 8$$

$$2^5 = 2 \text{ to the power } 5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

$$4^3 = 4 \text{ to the power } 3 = 4 \times 4 \times 4 = 64$$

When no power is written, power is 1

$$5 = 5 \text{ is same as } 5^1$$

$$9 = 9 \text{ is same as } 9^1$$

If you can purchase 5 maggie packets in 100 Rs then how much money you will need to purchase 10 maggie packets ?

**Multiplication**

$$2^3 \times 2^2$$

$$2^3 = 2 \times 2 \times 2, \quad 2^2 = 2 \times 2$$

$$2^3 \times 2^2 = 2 \times 2 \times 2 \times 2 \times 2$$

How many times we have 2 on right side of = ?

→ 5 times

$$\text{So, } 2^3 \times 2^2 = 2^5 \quad \text{here } 3 + 2 = 5$$

When we multiply same base number with some powers, we get answer as same base with addition of powers. So even if you get below formula, you can generate it yourself.

$$a^m \times a^n = a^{(m+n)}$$

$$3^4 \times 3^2 = 3^{(4+2)} = 3^6$$

$$5^7 \times 5^3 = 5^{(7+3)} = 5^{10}$$

$$9^2 \times 9^4 = 9^{(2+4)} = 9^6$$

$$8^4 \times 8^3 = 8^{(4+3)} = 8^7$$

### Division

$$2^6 \div 2^4$$

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2, \quad 2^4 = 2 \times 2 \times 2 \times 2$$

$$\frac{2^6}{2^4} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = 2 \times 2$$

How many times we have 2 on right side of = ?

→ 2 times

$$\text{So, } \frac{2^6}{2^4} = 2^2 \quad \text{here } 6 - 4 = 2$$

When we divide same base number with some powers we get answer as same base with subtraction of powers. So even if you get below formula, you can generate it yourself

$$a^m \div a^n = a^{(m-n)}$$

$$2^8 \div 2^3 = 2^{(8-3)} = 2^5$$

$$5^7 \div 5^3 = 5^{(7-3)} = 5^4$$

$$9^4 \div 9^2 = 9^{(4-2)} = 9^2$$

$$3^4 \div 3^3 = 3^{(4-3)} = 3^1$$



Power To Power

$$(3^4)^2$$

Here we know  $3^4 = 3 \times 3 \times 3 \times 3$

$$(3^4)^2 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

How many times we have 3 on right side of =??

→ 8 Times

$$(a^m)^n = a^{m \times n}$$

$$(2^3)^5 = 2^{3 \times 5} = 2^{15}$$

$$(5^2)^3 = 5^{2 \times 3} = 5^6$$

$$(7^4)^3 = 7^{4 \times 3} = 7^{12}$$

$$(8^3)^2 = 8^{3 \times 2} = 8^6$$

### Power of 0

Any Nonzero number to power 0 is always 1

$$a^0 = 1$$

$$8^0 = 1$$

$$78^0 = 1$$

$$23^0 = 1$$

$$8222^0 = 1$$

**Power of 1**

Any number to power 1 is always number itself

$$a^1 = a$$

$$8^1 = 8$$

$$78^1 = 78$$

$$23^1 = 23$$

$$8222^1 = 8222$$

**Negative power**

If you want to convert negative power to positive just shift it numerator to denominator or vice versa

$$a^{-m} = \frac{1}{a^m}$$

$$6^{-3} = \frac{1}{6^3}$$

$$5^{-4} = \frac{1}{5^4}$$

$$\frac{1}{6^{-3}} = 6^3$$

**Multiplication and division to same power**

$$1) (a \times b)^m = a^m \times b^m$$

We need to assign power m to a and b both and just keep same operation \* or /

$$(3 \times 5)^2 = 3^2 \times 5^2$$

$$1) (a / b)^m = a^m / b^m$$

We need to assign power m to a and b both and just keep same operation \* or /

$$(3 / 5)^2 = 3^2 / 5^2$$

**All Rules for Exponent/Power/Index**

$$a^m \times a^n = a^{(m+n)}$$

$$a^m \div a^n = a^{(m-n)}$$

$$(a^m)^n = a^{m \times n}$$

$$a^0 = 1$$

$$a^1 = a$$

$$a^{-m} = \frac{1}{a^m}$$

$$(a \times b)^m = a^m \times b^m$$

$$(a / b)^m = a^m / b^m$$

**Fraction****Real time example of Fraction**

Let us assume you bought 10 chocolates out of which you ate 7  
chocolates

Now if someone ask you how many chocolates you ate and how many are  
remaining, it is straight forward you count as natural number and say I ate  
7. How many remaining  $10 - 7 = 3$

This is very simple and anyone can tell it.

Now assume you bought 1 pizza which has 7 equal parts. Now you ate 4  
parts out of it. Now can you say how many pizzas you ate??? You can't say  
4 as original pizza was 1 only and how you can eat 4????

Here fraction comes into picture

How many parts were there originally?

7

How many you ate out of them??

4

So you can say you ate 4 parts out of 7 which we represent as  $4/7$

How many parts remaining??

3

Out of 7 parts, 3 parts are remaining

So  $3/7$  is remaining

If you buy pizza and explain this concept, I believe anyone will understand  
concept of Fraction very easily.

### Comparing Fractions

Most of us can easily compare Integers but face challenge to compare fractions

1) Which number is bigger 3 or 5???

Obviously 5 as we know  $3 < 5$  This is very simple

Reason for this is when denominator not mentioned it is 1

$$\frac{3}{1} < \frac{5}{1}$$

As denominator same we compare numerator only and decide

$$\frac{5}{7} < \frac{8}{7} \quad (\text{As } 5 < 8)$$

2) When denominator is not same we should just do cross multiplication

$$\begin{array}{ccc} 1 & & 5 \\ \hline 2 & & 3 \end{array}$$

$$\begin{array}{ccc} 1 \times 3 & & 5 \times 2 \\ 3 & < & 10 \end{array}$$

$$\frac{1}{2} < \frac{5}{3}$$

$$\begin{array}{cc} 5 & 3 \\ \hline 7 & 8 \end{array}$$

$$\begin{array}{cc} 5 \times 8 & 3 \times 7 \\ 40 & > 21 \end{array}$$

$$\frac{5}{7} > \frac{3}{8}$$

### Comparing more than 2 Fractions

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$$

As denominator not same comparing numerator wont work

How denominator can be equal??

Take LCM of all denominators

Also find by which number you should multiply each denominator you get

12, multiply respective numerator by same number

$$\begin{array}{ccc} 1 \times 6 & 2 \times 4 & 3 \times 3 \\ \hline 2 \times 6 & 3 \times 4 & 4 \times 3 \end{array}$$

$$\frac{6}{12} < \frac{8}{12} < \frac{9}{12}$$

$$\frac{1}{2} < \frac{2}{3} < \frac{3}{4}$$

**Addition of 2 fractions:**

1) When denominator is same Keep denominator as is and add numerators

$$\frac{5}{7} + \frac{3}{7} = \frac{5+3}{7} = \frac{8}{7}$$

$$\frac{8}{3} + \frac{5}{3} = \frac{8+5}{3} = \frac{13}{3}$$

2) When denominator not same, cross multiply

$$\frac{1}{2} + \frac{5}{3}$$

$$\frac{1}{2} + \frac{5}{3} = \frac{(1 \times 3) + (5 \times 2)}{(2 \times 3)} = \frac{3 + 10}{6}$$

$$= \frac{13}{6}$$

**Addition of more than 2 fractions**

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$$

As denominator not same addition of numerator wont work

How denominator can be equal??

Take LCM of all denominators

Also find by which number you should multiply each denominator you get 12, multiply respective numerator by same number

$$\frac{1 \times 6}{2 \times 6}, \quad \frac{2 \times 4}{3 \times 4}, \quad \frac{3 \times 3}{4 \times 3}$$

$$\frac{6}{12} + \frac{8}{12} + \frac{9}{12} = \frac{23}{12}$$

**Subtraction of 2 fractions:**

1) When denominator is same Keep denominator as is and add numerators

$$\frac{5}{7} - \frac{3}{7} = \frac{5-3}{7} = \frac{2}{7}$$

$$\frac{8}{3} - \frac{5}{3} = \frac{8-5}{3} = \frac{3}{3} = 1$$



2) When denominator not same, cross multiply

$$\frac{5}{3} - \frac{1}{2}$$

$$\frac{5}{3} - \frac{1}{2} = \frac{(5 \times 2) - (1 \times 3)}{(3 \times 2)} = \frac{10 - 3}{6}$$

$$= \frac{7}{6}$$

$$\text{Anay} = \frac{78}{50} \times 100 = 78\%$$

### Multiplication of Fractions

Multiplication is simplest operations for fractions

$$\frac{5}{3} \times \frac{1}{2} = \frac{\text{Multiply all numerators } 5 \times 1}{\text{Multiply all denominators } 3 \times 2} = \frac{5}{6}$$

$$\frac{3}{5} \times \frac{7}{8} = \frac{3 \times 7}{5 \times 8} = \frac{21}{40}$$

$$\frac{10}{3} \times \frac{7}{20} \times \frac{9}{14} = \frac{10 \times 7 \times 9}{3 \times 20 \times 14} = \frac{630}{840} = \frac{21}{28} = \frac{3}{4}$$

This causes big number. Instead simplify first and then solve

$$\frac{10}{3} \times \frac{7}{20} \times \frac{9}{14} = \frac{1}{3} \times \frac{1}{2} \times \frac{3}{2} \quad [\text{Simplified 7 and 14, 10 and 20, 3 and 9}]$$

$$\frac{3}{4}$$

### **Reciprocal:**

How do you stand normally?

### **Head Upwards**

### **Leg Downwards**

Now imagine you are standing on your head

### **Leg Upwards**

### **Head Downwards**

This is nothing but reciprocal. If we interchange numerator and denominator we get reciprocal (sign need not to be changed)

### **Find Reciprocals**

$$1) \frac{1}{7} \rightarrow \frac{7}{1} = 7$$

$$2) \frac{-4}{11} \rightarrow \frac{-11}{4}$$

**Fractions Division:**

$$\frac{7}{2}$$

---

$$\frac{1}{3}$$

Keep first fraction as is

Get reciprocal of 2<sup>nd</sup> fraction

Multiply both of them

Keep 7/2 as is

Reciprocal of 1/3 is 3

Multiply them

$$\frac{7}{2} \times \frac{3}{1} = \frac{21}{2}$$

**Finding Equivalent fractions**

Equivalent means equal (=)

If we multiply any number by 1 we get same number So we say it is equivalent number

$$2 \rightarrow 2 \times 1 = 2, \quad 10 \rightarrow 10 \times 1 = 10$$

Can you imagine different ways of showing 1 as fraction???

$$\rightarrow \frac{2}{2}, \frac{3}{3}, \frac{5}{5}, \frac{10}{10} \text{ etc.}$$

Find 3 equivalent fraction of  $\frac{3}{5}$

$$\begin{aligned} \rightarrow \frac{3}{5} \times \frac{2}{2} &= \frac{6}{10} \\ \frac{3}{5} \times \frac{10}{10} &= \frac{30}{50} \\ \frac{3}{5} \times \frac{5}{5} &= \frac{15}{25} \end{aligned}$$

### Finding fractions between given range

1) Find fractions between  $\frac{2}{7}$  and  $\frac{9}{7}$

→ It is very simple as denominator is same

$$\frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}, \frac{8}{7}$$

2) Find 20 fractions between  $\frac{2}{7}$  and  $\frac{4}{7}$

→ Denominator same but we can see only 1 fraction that is  $\frac{3}{7}$

→ If we want more fractions between given range, we should have big difference in numerator and so big denominator

Get equivalent fraction by multiplying by big number like 50

$$\begin{aligned} 2 \times 50 &= 100 \\ \frac{100}{7 \times 50} &= \frac{100}{350} \end{aligned}$$

$$\begin{aligned} 4 \times 50 &= 200 \\ \frac{200}{5 \times 50} &= \frac{200}{350} \end{aligned}$$

Everything from  $\frac{101}{350}$  to  $\frac{199}{350}$  will come between  $\frac{2}{7}$  and  $\frac{4}{7}$

### Converting to Simplest form

Simplest form means keep dividing numerator and denominator by common factor till largest common factor is 1

$\frac{32}{48}$  when divided by 2  $\frac{16}{24} \rightarrow \frac{8}{12} \rightarrow \frac{4}{6} \rightarrow \frac{2}{3}$

Instead you can find HCF of Numerator and denominator and divide both of them by it

HCF of 32 and 48 is 16

$\frac{32}{48} \div \frac{16}{16} = \frac{2}{3}$

## HCF - LCM

**Real time Usage of HCF:**

1) Assume there is milkman who supplies milk to 2 different companies ABC and XYZ. Every day he needs to supply 60 litres Milk to ABC and 80 litres milk to XYZ. Now he has huge tank in which milk is already stored. He need to buy container by which he can measure 60 litres as well as 80 litres. He can afford to buy only 1 container.

How much capacity container he should buy?

**Solution:** He should find HCF of 60 and 80. Buy container of HCF size.

2) There are 50 Boys and 60 Girls in class. Teacher needs to arrange them in different rows such way that

a) Every row should have same number of students.

b) Each row there should be either all boys or all girls (No combination of boy and girl in any row).

c) Minimum number of rows should be used so that Teacher's voice can reach last student comfortably.

**Solution:** Teacher should find HCF of 50 and 60. Keep those many students in each row.

**Real time example of LCM**

1) Nikita and Monika usually do running practice on playground in their area. Playground is Circular in shape. Starting from start point and again reaching at same start point is 1 round. Nikita takes 15 minutes for 1 round and Monika takes 20 minutes for 1 round. They decided that when they meet at start point together, they will stop running and go home. Both of them started running together from start point at 8:00 AM. When they will stop running?

**Solution:** They should find LCM of 15 and 20. After those many minutes they should stop.

2) Krutika and Ajaya are very good friend. They want to plan 1 day trip.

However, both are working and they have different holiday schedule.

Staying from 1<sup>st</sup> day of Month:

Krutika's company. She needs to work continuous 5 days and holiday on

6<sup>th</sup>

Ajay's company: He needs to work continuous 4 days and holiday on 5<sup>th</sup>  
This cycle repeats till end of month

It is December month right now Which date of December is perfect for both of them for trip (Both should have holiday on same day).

**Solution:** They should find LCM of 6 and 5 That date will be perfect for trip

HCF(Highest Common Factor) and LCM(Lowest Common Multiple)

Find HCF and LCM of 50 and 60

Step 1: Do Prime factorization for both numbers

Step 2: Write each number along with occurrence(number of times) of it

Step 3: HCF: Take less number of times count

LCM: Take more number of times for LCM

Step 1: Do Prime factorization for both numbers

$$50 = 2 \times 25$$

$$60 = 2 \times 30$$

$$50 = 2 \times 5 \times 5$$

$$= 2 \times 2 \times 15$$

$$50 = 2 \times 5 \times 5$$

$$60 = 2 \times 2 \times 3 \times 5$$

Step 2: Write each number along with occurrence(number of times) of it

2: 1 Time

2: 2 Times

5: 2 Times

5: 1 Time

3: 0 Time

3: 1 Time

Step 3: HCF: Take less number of times count

$$2 \times 5 = 10$$

LCM: Take more number of times for LCM

$$2 \times 2 \times 5 \times 5 \times 3 = 300$$

### Finding HCF Orally:

HCF means Highest Common Factor In Simple words, biggest number by which given numbers are completely divisible.

### Steps to find HCF Orally:

Step 1: Keep smaller number as is

Step 2: Find Bigger number – smaller number

Step 3: Try to find HCF of Step1 answer and Step2 answer orally

If you can't, repeat Step 1 to Step 3 till you get same answer for  
Step 1 and Step 2

**Problem 1: 40, 60**

Step 1: Smaller number = 40

Step 2: Bigger number – smaller number =  $60 - 40 = 20$

Step 3: Find HCF of 40 and 20

Step 1: Smaller number = 20

Step 2: Bigger number – smaller number =  $40 - 20 = 20$

Step 3: HCF of 20 and 20 = 20

**HCF of 40 and 60 is 20**

**Problem 2: 24, 72**

Step 1: Smaller number = 24

Step 2: Bigger number – smaller number =  $72 - 24 = 48$

Step 3: Find HCF of 24 and 48

Step 1: Smaller number = 24

Step 2: Bigger number – smaller number =  $48 - 24 = 24$

Step 3: HCF of 24 and 24 = 24

**HCF of 24 and 72 is 24**

**Finding LCM Orally:**

LCM means Least Common Multiple. In Simple words, Smallest number which comes in table of given numbers is nothing but LCM.

Step 1: Identify biggest number

Step 2: Start writing table of it (Take help of addition)

If number coming in table is divisible by other numbers, stop. It is answer of LCM

**Problem 1: 20, 30**

Step 1: Bigger number = 30

Step 2: 30 (Not divisible by 20)

60 (Divisible by 20)

Stop. 60 is LCM

**LCM of 20 and 30 is 60**



**Problem 2: 45, 30**

Step 1: Bigger number = 45

Step 2: 45 (Not divisible by 30)

90 (Divisible by 30)

Stop. 90 is LCM

**LCM of 45 and 30 is 90**

**Problem 3: 12, 28**

Step 1: Bigger number = 28

Step 2: 28 (Not divisible by 12)

56 (Not Divisible by 12)

84 (Divisible by 12)

Stop. 84 is LCM

**LCM of 12 and 28 is 84**

**Problem 4: 20, 30, 40**

Step 1: Bigger number = 40

Step 2: 40 (Divisible by 20 but not by 30)

80 (Divisible by 20 but not 30)

120 (Divisible by 20 and 30 both)

Stop. 120 is LCM

**LCM of 20, 30 and 40 is 120**

**Verification of Answers of HCF and LCM**

**Find HCF and LCM of 10 and 18. Also verify your answer**

HCF of 10, 18

Step 1: Smaller number = 10

Step 2: Bigger number – smaller number =  $18 - 10 = 8$

Step 3: Find HCF of 8 and 10

Step 1: Smaller number = 8

Step 2: Bigger number – smaller number =  $10 - 8 = 2$

Step 3: Find HCF of 2 and 8

Every even number is divisible by 2

HCF of 2 and 8 is 2

**HCF of 10 and 18 is 2**

**LCM of 10 and 18**

Step 1: Bigger number = 18

Step 2: 18 (Not divisible by 10)

36 (Not divisible by 10)

54 (Not divisible by 10)

72 (Not divisible by 10)

90 (Divisible by 10)

Stop 90 is LCM

**LCM of 10 and 18 is 90**

For any 2 numbers, their product is always equal to HCF X LCM

$1^{\text{st}} \text{ Number} \times 2^{\text{nd}} \text{ Number} = \text{HCF} \times \text{LCM}$

$$10 \times 18 = 2 \times 90$$

$$180 = 180$$

## Mixture or Alligation

### Real time Usage of Mixture or Alligation:

Kasturi decided to give small party to her friends. She decided to give Mango juice to all friends at home.

She went to Shop for purchasing Mangoes. Shopkeeper told there are 2 categories:

Category 1: 100 Rs per Kg

Category 2: 500 Rs per Kg

Kasturi thought Category 1 would be very cheap and Category 2 will be very expensive.

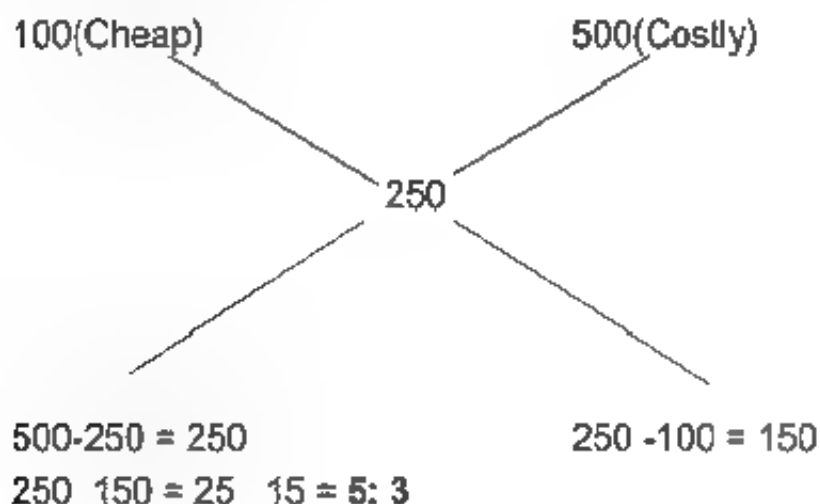
She asked shopkeeper that give combination of both categories such a way that it will cost her 250 Rs per kg.

Shopkeeper was confused. At same time 1 mathematician came there.

Kasturi and Shopkeeper asked same question to him.

Mathematician says its SIMPLE and we call it as **Alligation**

- 1) Draw BIG X
- 2) Write cheaper value at left top
- 3) Write costly value at right top
- 4) Write number which you want to get at intersection of X
- 5) Subtract diagonally to get ratio



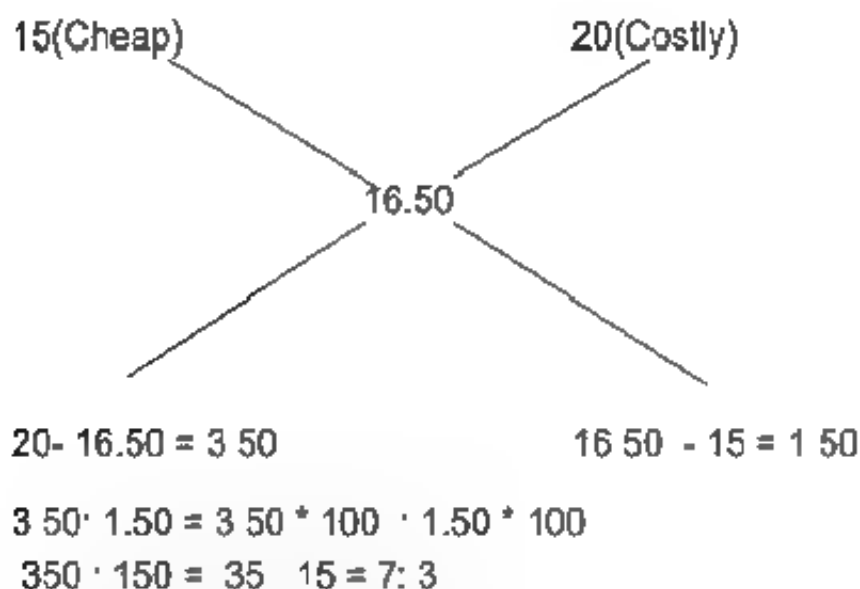
**Shopkeeper should mix Category 1 and Category 2 in ratio of 5: 3**

**Problem 1:**

In what ratio must a grocer should mix 2 varieties of wheat costing 15 Rs per kg and 20 Rs per kg so as to get mixture worth 16.50 Rs per kg?

**Solution:**

- 1) Draw BIG X
- 2) Write cheaper value at left top
- 3) Write costly value at right top
- 4) Write number which you want to get at intersection of X
- 5) Subtract diagonally to get ratio



**Grocer should mix varieties in ratio of 7: 3**

**Problem 2:**

In what ratio must 62 Rs per kg rice mixed with 72 Rs per kg to obtain mixture worth 64.50 Rs?

**Solution:**

- 1) Draw BIG X
- 2) Write cheaper value at left top
- 3) Write costly value at right top
- 4) Write number which you want to get at intersection of X
- 5) Subtract diagonally to get ratio

$$\begin{array}{ccc}
 62(\text{Cheap}) & & 72(\text{Costly}) \\
 & \searrow \quad \swarrow & \\
 & 64.50 & \\
 & \swarrow \quad \searrow & \\
 72 - 64.50 = 7.50 & & 64.50 - 62 = 2.50 \\
 7.50 \cdot 2.50 = 7.50 \cdot 100 \cdot 2.50 \cdot 100 & & \\
 750 \cdot 250 = 75 \cdot 25 = 3:1 & & 
 \end{array}$$

Let us verify our answer

$$62 \text{ Rs per kg for 3 Kg} = 62 \cdot 3 = 186 \text{ Rs}$$

$$72 \text{ Rs per kg for 1 Kg} = 72 \cdot 1 = 72 \text{ Rs}$$

-----

$$258 \text{ Rs}$$

$$64.50 \text{ per kg for 4 kg (3+1)}$$

$$64.50 \cdot 4 = 258 \text{ Rs}$$

As left side and right side amount matching we can say our answer is verified and correct

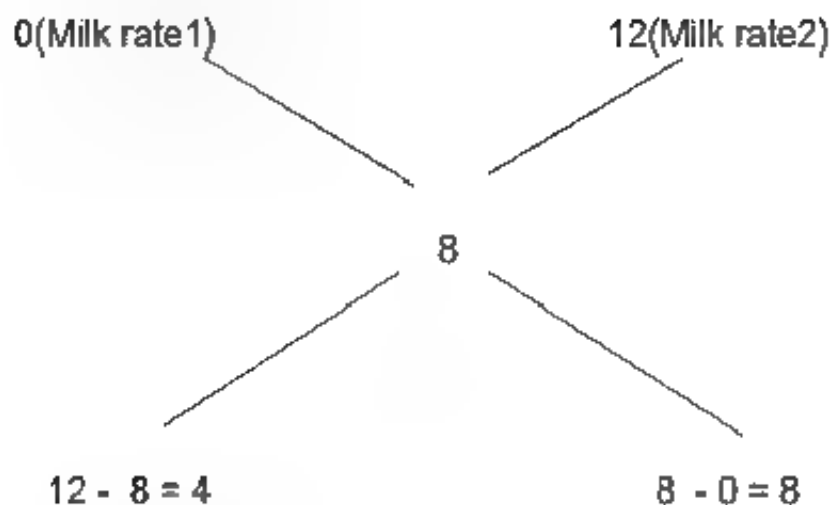
**Rice should be mixed in ratio of 3: 1**

### Problem 3:

In what ratio must water be mixed with milk costing 12 Rs per litre to obtain mixture worth of 8 Rs per litre.

### Solution:

Here we are trying to mix Water with Milk to sell it as milk. We can say 0Rs for water which we are selling as milk



$$\text{Water : Milk} = 4 : 8 = 1 : 2$$

**Verify :**

1 litre Water = 0 Rs

Total mixture =  $8 \times 3 = 24$  Rs

2 litres Milk =  $12 \times 2 = 24$  Rs

As left side amount and right side amount matching we can say our answer is verified and correct

**Water: Milk ratio should be 1: 2**

**Problem 4:**

The milk and water in two vessels A and B are in ratio 4: 3 and 2: 3 respectively. In what ratio liquids in both vessels to be mixed to obtain new mixture in vessel C containing half milk and half water?

**Solution:**

Vessel A: Milk:Water = 4 : 3

This means if there are 4 parts of milk, there are 3 parts of water

Total parts = 7 [4 + 3 = 7]

$$\text{Milk Fraction} = \frac{\text{Milk Parts}}{\text{Total Par}} = \frac{4}{7}$$

Vessel B: Milk:Water = 2 : 3

This means if there are 2 parts of milk, there are 3 parts of water

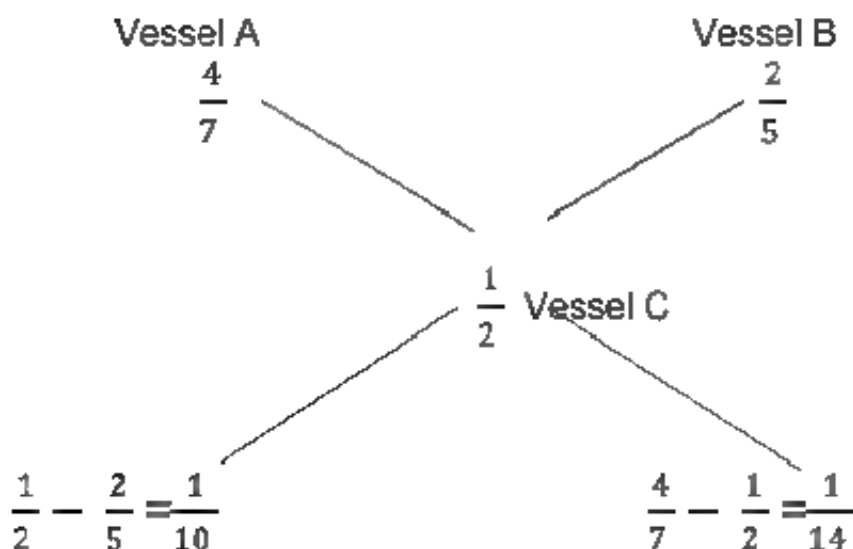
Total parts = 5 [2 + 3 = 5]

$$\text{Milk Fraction} = \frac{\text{Milk Parts}}{\text{Total Parts}} = \frac{2}{5}$$

Vessel C: We want same milk and water [Half mentioned]

If 1 part of Milk , 1 part of Water

$$\text{Milk Fraction} = \frac{\text{Milk Parts}}{\text{Total Part}} = \frac{1}{2}$$



$$\frac{1}{10} : \frac{1}{14}$$

LCM of 10 and 14 is 70. So multiply both fractions by 70

$$\frac{1}{10} \times \frac{70}{1} = 7$$

$$\frac{1}{14} \times \frac{70}{1} = 5$$

**7: 5**

Vessel A and Vessel B should be mixed in ratio of 7:5

**Problem 5:**

Two vessels A and B containing spirit and water mixed in ratio of 5:2 and 7:6 respectively. Find ratio in which these mixtures will be mixed to obtain new mixture in vessel C containing Spirit and water ratio 8:5

**Solution:**

Vessel A: Spirit : Water = 5 : 2

This means if there are 5 parts of spirit, there are 2 parts of water

Total parts = 7 [5 + 2 = 7]

$$\text{Spirit Fraction} = \frac{\text{Spirit Parts}}{\text{Total Parts}} = \frac{5}{7}$$

Vessel B: Spirit : Water = 7 : 6

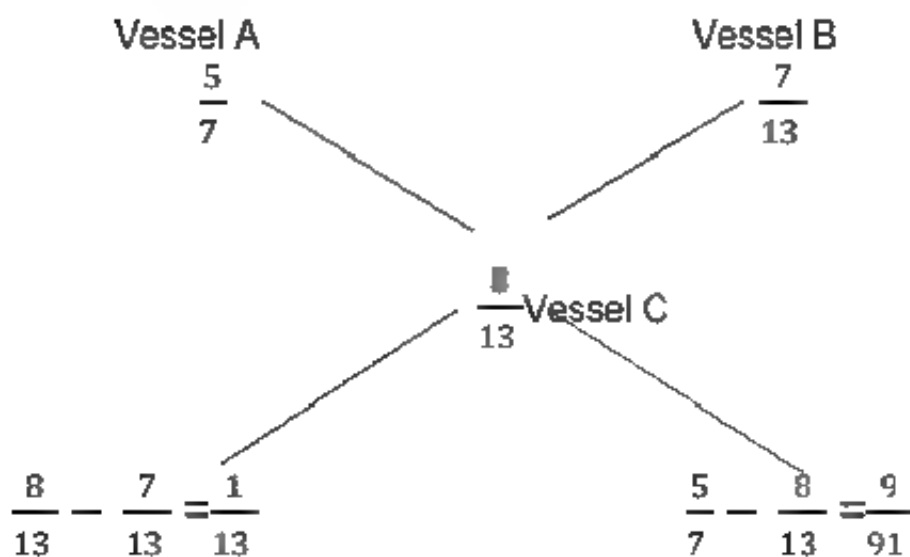
This means if there are 7 parts of spirit, there are 6 parts of water

Total parts = 13 [7 + 6 = 13]

$$\text{Spirit Fraction} = \frac{\text{Spirit Parts}}{\text{Total Parts}} = \frac{7}{13}$$

Vessel C: We want Spirit : Water ratio as 8 : 5

$$\text{Spirit Fraction} = \frac{\text{Spirit Parts}}{\text{Total Parts}} = \frac{8}{8+5} = \frac{8}{13}$$





$$\frac{1}{13} : \frac{9}{91}$$

LCM of 13 and 91 is 91. So multiply both fractions by 91

$$\frac{1}{13} \times \frac{91}{1} = 7$$

$$\frac{9}{91} \times \frac{91}{1} = 9$$

**7: 9**

**Vessel A and Vessel B should be mixed in ratio of 7 : 9**

**Percentage****Real time example of Percentage**

In Simple words, Percentage means per 100.

Why we need to learn Percentage?

Ruchita got 45 marks out of 50 and Anay got 78 marks out of 100 Who got better marks?

→ We should find Percentage for both of them and then only we can conclude who got better marks

$$\text{Ruchita} = \frac{45}{50} \times 100 = 90\%$$

$$\text{Anay} = \frac{78}{100} \times 100 = 78\%$$

Now we can conclude Ruchita got better marks as 90 is bigger than 78.

**Percentage – Convert Fraction to %****Problem 1:**

$$\frac{1}{2} = \frac{1}{2} \times 100 = 50\%$$

**Problem 2:**

$$\frac{4}{5} = \frac{4}{5} \times 100 = 4 \times 20 = 80\%$$

**Problem 3:**

$$\frac{8}{5} = \frac{8}{5} \times 100 = 8 \times 20 = 160\%$$

**Percentage – Convert Ratio to %****Problem 1:**

$$1:5 = \frac{1}{5} \times 100 = 20\%$$

**Problem 2:**

$$7:10 = \frac{7}{10} \times 100 = 70\%$$

**Problem 3:**

$$9:4 = \frac{9}{4} \times 100 = 9 \times 25 = 225\%$$

**Convert Decimal to %**

Before learning this, let us understand multiplication of decimal with 10, 100  
Whenever we multiply any decimal number by 10, decimal gets shifted to right by 1 digit (as 10 has 1 zero)

$$0.63 \times 10 = 6.3$$

$$0.345 \times 10 = 3.45$$

Whenever we multiply any decimal number by 100, decimal gets shifted to right by 2 digits (as 100 has 2 zeros)

**Convert following decimal to %:****Problem 1:**

$$0.89 = 0.89 \times 100 = 89\%$$

**Problem 2:**

$$6.734 = 6.734 \times 100 = 673.4\%$$

**Problem 3:**

$$5.21 = 5.21 \times 100 = 521\%$$

**Convert % to Fraction**

We just need to divide given percentage value by 100 to get it converted into Fraction. Convert Fraction into Simplest form [Simplest form means keep dividing till HCF of Numerator and Denominator as 1]

**Problem 1:**

$$50\% = \frac{50}{100} = \frac{1}{2}$$

**Problem 2:**

$$80\% = \frac{80}{100} = \frac{8}{10} = \frac{4}{5}$$

**Problem 3:**

$$160\% = \frac{160}{100} = \frac{16}{10} = \frac{8}{5}$$

**Problem 4:**

$$900\% = \frac{900}{100} = \frac{9}{1}$$

**Convert % to Ratio**

We just need to divide given percentage value by 100 to get it converted into Fraction. Convert Fraction into Simplest form and show it as Ratio

**Problem 1:**

$$88\% = \frac{88}{100} = \frac{44}{50} = \frac{22}{25} = 22 : 25$$

**Problem 2:**

$$320\% = \frac{320}{100} = \frac{32}{10} = \frac{16}{5} = 16 : 5$$

**Problem 3:**

$$78\% = \frac{78}{100} = \frac{39}{50} = 39 : 50$$

**Convert % to Decimal**

We just need to divide given percentage value by 100 to get it converted into Decimal

When we divide any number by 100, decimal gets shifted to left by 2 places

**Problem 1:**

$$84\% = \frac{84}{100} = 0.84$$

**Problem 2:**

$$225\% = \frac{225}{100} = 2.25$$

When we don't see any decimal in given number it is after right most digit.

Example 73 same as 73.0

**Find some% of Something****Problem 1:**

$$10\% \text{ of } 8000 = \frac{10}{100} \times 8000 = 10 \times 80 = 800$$

**Problem 2:**

$$20\% \text{ of } 1600 = \frac{20}{100} \times 1600 = 20 \times 16 = 320$$

**Problem 3:**

$$45\% \text{ of } 400 = \frac{45}{100} \times 400 = 45 \times 4 = 180$$

**Problem 4:**

$$55\% \text{ of } 6000 = \frac{55}{100} \times 6000 = 55 \times 60 = 3300$$

**Problem 5:**

$$18\% \text{ of } 220 = \frac{18}{100} \times 220 = \frac{396}{10} = 39.6$$

**Shortcut for Calculating Standard %**

$$1) 5\% = \frac{5}{100} = \frac{1}{20}$$

If we have to find 5% of any number just divide given number by **20**

$$5\% \text{ of } 500 = \frac{500}{20} = 25$$

$$2) 10\% = \frac{10}{100} = \frac{1}{10}$$

If we have to find 10% of any number just divide given number by **10**

$$10\% \text{ of } 1000 = \frac{1000}{10} = 100$$

$$3) 20\% = \frac{20}{100} = \frac{1}{5}$$

If we have to find 20% of any number just divide given number by **5**

$$20\% \text{ of } 600 = \frac{600}{5} = 120$$

$$4) 25\% = \frac{25}{100} = \frac{1}{4}$$

If we have to find 25% of any number just divide given number by **4**

$$25\% \text{ of } 800 = \frac{800}{4} = 200$$

$$5) 50\% = \frac{50}{100} = \frac{1}{2}$$

If we have to find 50% of any number just divide given number by **2**

$$50\% \text{ of } 900 = \frac{900}{2} = 450$$

**Calculate % in Tens Orally / Quickly**

1) 30 % of 800

Finding 10% of any number just divide given number by **10**

$$10\% \text{ of } 800 = \frac{800}{10} = 80$$

$$\text{As } 30 = 10 \times 3, 80 \times 3 = \mathbf{240}$$

2) 40 % of 1700

Finding 10% of any number just divide given number by **10**

$$10\% \text{ of } 1700 = \frac{1700}{10} = 170$$

$$\text{As } 40 = 10 \times 4, 170 \times 4 = \mathbf{680}$$

3) 60 % of 1200

Finding 10% of any number just divide given number by **10**

$$10\% \text{ of } 1200 = \frac{1200}{10} = 120$$

$$\text{As } 60 = 10 \times 6, 120 \times 6 = \mathbf{720}$$

4) 80 % of 900

Finding 10% of any number just divide given number by **10**

$$10\% \text{ of } 900 = \frac{900}{10} = 90$$

$$\text{As } 80 = 10 \times 8, 90 \times 8 = \mathbf{720}$$

5) 90 % of 8000

Finding 10% of any number just divide given number by **10**

$$10\% \text{ of } 8000 = \frac{8000}{10} = 800$$

$$\text{As } 90 = 10 \times 9, 800 \times 9 = \mathbf{7200}$$

**Calculate % Orally / Quickly**

Find below % of 800

a)4%   b)7%   c)13%   d)24%   e)38%   f)62%

**Solution:**

Here all percentage which are asked are non tens, basically no ending with 0. In this case we can see calculating 1% is easy???

1% means dividing given number by 100 or shifting decimal to 2 digits left

$$1\% \text{ of } 800 = \frac{800}{100} = 8$$

Just multiply that answer by required %

$$a)4\% = 4 \times 8 = 32$$

$$b)7\% = 7 \times 8 = 56$$

$$c)13\% = 13 \times 8 = 104$$

$$d)24\% = 24 \times 8 = 192$$

$$e)38\% = 38 \times 8 = 304$$

$$f)62\% = 62 \times 8 = 496$$

**Find new value after % increase****Problem 1:**

8000 Rs increased by 20%

**Solution**

First calculate 20% of 8000 and then add it to 8000

$$\frac{20}{100} \times 8000 = 1600$$

$$8000 + 1600 = 9600 \text{ Rs}$$

Shortcut for same

Increase in 20% means if original value was 100 it should become 120



Original      new

100 ----- 120

8000-----x

Cross multiply

$$100 \times x = 120 \times 8000$$

$$x = \frac{120}{100} \times 8000 = \mathbf{9600Rs}$$

### Problem 2:

450 Kg increased by 60%

### Solution

First calculate 60% of 450 and then add it to 450

$$\frac{60}{100} \times 450 = 270$$

$$450 + 270 = \mathbf{720 \text{ Kg}}$$

Shortcut for same

Increase in 60% means if original value was 100 it should become 160

Original      new

100 ----- 160

450-----x

Cross multiply

$$100 \times x = 160 \times 450$$

$$x = \frac{160}{100} \times 450 = \mathbf{720 \text{ Kg}}$$

### Find new value after % decrease

### Problem 1:

8000 Rs decreased by 20%

### Solution

First calculate 20% of 8000 and then subtract it from 8000

$$\frac{20}{100} \times 8000 = 1600$$

$$8000 - 1600 = \mathbf{6400 \text{ Rs}}$$

Shortcut for same

Decrease in 20% means if original value was 100 it should become 80

Original      new

100 -----80

8000-----x

Cross multiply

$$100 \times x = 80 \times 8000$$

$$x = \frac{80}{100} \times 8000 = \mathbf{6400 \text{ Rs}}$$

### Problem 2:

450 Kg decreased by 60%

### Solution

First calculate 60% of 450 and then subtract it from 450

$$\frac{60}{100} \times 450 = 270$$

$$450 - 270 = \mathbf{180 \text{ Kg}}$$

Shortcut for same

Decrease in 60% means if original value was 100 it should become 40

Original      new

100 -----40

450-----x

Cross multiply

$$100 \times x = 40 \times 450$$

$$x = \frac{40}{100} \times 450 = \mathbf{180 \text{ Kg}}$$

**Word Problems****Problem 1**

There are 5000 plants in nursery 5% plants are of rose, 1% are of mango  
What is total number of other plants?

**Solution:**

5% plants are of rose

$$\frac{5}{100} \times 5000 = 250 \text{ plants are of rose}$$

1% plants are of mango

$$\frac{1}{100} \times 5000 = 50 \text{ plants are of mango}$$

$$\text{Rose} + \text{Mango} = 250 + 50 = 300$$

$$\begin{aligned} \text{Other plants} &= \text{Total} - (\text{Rose} + \text{Mango}) \\ &= 5000 - 300 = 4700 \end{aligned}$$

**Other plants are 4700**

**Problem 2**

Reema got 98 marks in her exam This was 56% of total marks. Find what are maximum marks?

**Solution:**

As we don't know what are maximum marks, let us assume maximum marks as x

Reema got 98 marks which is 56% of x

$$\frac{56}{100} \times x = 98$$

$$x = \frac{98 \times 100}{56} = 175$$

**Maximum marks are 175**

**Problem 3**

X is 5% of Y, Y is 24% of Z. If X = 480 find value of Y and Z

**Solution:**

$$X = \frac{5}{100} \times Y$$

$$480 = \frac{5}{100} \times Y$$

$$Y = \frac{480 \times 100}{5} = 9600$$

$$Y = \frac{24}{100} \times Z$$

$$9600 = \frac{24}{100} \times Z$$

$$Z = \frac{9600 \times 100}{24} = 40000$$

**Y value is 9600 and Z value is 40000**

**Problem 4**

Rahul's income is 25% more than Rohan. What percent of Rohan's income less than Rahu's income?

**Solution:**

Here by looking a question if one person having 25% more than other then other person is having 25% less than. It is not like this

Let us assume Rohan's income = 100 Rs

Rahul's income = 100 + 25 = 125 Rs

We can say Rohan's income 25 Rs less than Rahu's income

$$\% \text{ less income} = \frac{25}{125} \times 100 = 20\%$$

Rohan's income is 20% less than Rahul's income

**Problem 5:**

Ankita got increment of 10% on her salary. Her new salary is 3575 Rs. What was her salary before increment?

**Solution:**

If earlier salary 100 then new salary is 110 Rs

$$100 \text{-----} 110$$

$$x \text{-----} 3575$$

Cross multiply

$$100 \times 3575 = 110 \times x$$

$$x = \frac{100 \times 3575}{110} = 3250 \text{ Rs}$$

**Ankita's original salary was 3250 Rs**

**Problem 6:**

Price of sugar increased by 20%. How much % we should reduce consumption of sugar so that our sugar expenditure remains same?

**Solution:**

Let us assume we were consuming 100 Kg sugar every month at 100 Rs per kg

$$\text{Our expenditure} = 100 \times 100 = 10000 \text{ Rs}$$

$$\text{New price of sugar} = 120 \text{ Rs per kg}$$

$$\text{In same amount we will get} = 10000/120 = 250/3 \text{ kg sugar only}$$

$$\text{Reduction in consumption} = \text{original} - \text{new}$$

$$= 100 - 250/3 = 50/3 = 16 \frac{2}{3}\%$$

**We should reduce consumption of sugar by  $16 \frac{2}{3}\%$**

## Probability

### Real time example of Probability

Probability means chance of certain thing to Happen

Assume if you give any exam What are possible results?

1) You may pass or 2) You may fail

Total Possibilities = 2

How many out of them favour Pass = 1

How many out of them favour Fail = 1

$$\text{Probability of Pass} = \frac{1}{2}$$

$$\text{Probability of Fail} = \frac{1}{2}$$

$$\text{Probability} = \frac{\text{Number of Possibilities in which given condition is satisfied}}{\text{Number of All Possibilities}}$$

### Probability - Coins

1 Coin:

Possibilities 1) Head 2) Tail

Total Possibilities : 2 [  $2^1 = 2$  ]

#### Problem 1:

If 1 Coin is tossed, what is probability of

a) Getting Head

Total Possibilities = 2

Favourable to Head = 1

$$\text{Probability of Getting Heads} = \frac{1}{2}$$

b) Getting neither head nor tail

Total Possibilities = 2

Favourable to neither head nor tail = 0

$$\text{Probability of Getting Heads} = \frac{0}{2} = 0$$

Probability of getting neither head nor tail = 0

c) Getting either head or tail

Total Possibilities = 2

Favourable to either head or tail = 2

$$\text{Probability of Getting Heads} = \frac{2}{2} = 1$$

Probability of getting either head or tail = 1

## 2 Coins:

Possibilities

| Sr No | 1 <sup>st</sup> Coin | 2 <sup>nd</sup> Coin |
|-------|----------------------|----------------------|
| 1     | H                    | H                    |
| 2     | H                    | T                    |
| 3     | T                    | H                    |
| 4     | T                    | T                    |

Total Possibilities = 4 [  $2^2 = 4$  ]

### Problem 1:

If 2 coins are tossed, what is probability of

a) Getting No heads:

Total Possibilities = 4

Favourable to getting no heads = 1 [TT]

$$\text{Probability of Getting Heads} = \frac{1}{4}$$

b) Getting exact 1 heads:

Total Possibilities = 4

Favourable to getting exact 1 heads = 2 [HT, TH]

$$\text{Probability of Getting Heads} = \frac{2}{4} = \frac{1}{2}$$

c) Getting at least 1 heads:

Total Possibilities = 4

At least 1 heads means 1,2,3... heads

Favourable to at least 1 heads = 3 [HH,HT,TH]

Probability of at least 1 heads =  $\frac{3}{4}$

### 3 Coins:

Possibilities:

| Sr No | 1 <sup>st</sup> Coin | 2 <sup>nd</sup> Coin | 3 <sup>rd</sup> Coin |
|-------|----------------------|----------------------|----------------------|
| 1     | H                    | H                    | H                    |
| 2     | H                    | H                    | T                    |
| 3     | H                    | T                    | H                    |
| 4     | H                    | T                    | T                    |
| 5     | T                    | H                    | H                    |
| 6     | T                    | H                    | T                    |
| 7     | T                    | T                    | H                    |
| 8     | T                    | T                    | T                    |

Total Possibilities = 8 [  $2^3 = 8$  ]

### Problem 1:

If 3 coins are tossed, what is probability of

a) Getting same outcome on all coins:

Total Possibilities = 8

Favourable to Getting same outcome on all coins:

= 2 [HHH, TTT]

Probability of Getting same outcome on all coins =  $\frac{2}{8} = \frac{1}{4}$

b) Getting at most 2 Heads

Total Possibilities = 8

At most 2 Heads means 0, 1, 2 heads

Favourable to Getting at most 2 Heads = 7 [All except HHH]

Probability of Getting at most 2 Heads =  $\frac{7}{8}$



**Dice:**

Everyone plays different games in which we use dice. However, when someone ask what is probability of getting 6, we get confused

### **1 Die**

Total Possibilities = [1, 2, 3, 4, 5, 6] = 6 [As  $6^1 = 6$ ]

#### **Problem 1:**

If 1 die is thrown what is probability of

a) Getting 6

Total Possibilities = 6

Favourable to Getting 6' = 1 [6]

Probability of Getting 6 =  $\frac{1}{6}$

b) Getting odd number

Total Possibilities = 6

Favourable to getting odd number' = 3 [1, 3, 5]

Probability of getting odd number =  $\frac{3}{6} = \frac{1}{2}$

c) Not getting 6

Total Possibilities = 6

Favourable to not getting 6' = 5 [1, 2, 3, 4, 5]

Probability of not getting 6 =  $\frac{5}{6}$

### **2 Dice**

Total Possibilities

1<sup>st</sup> number in bracket represent 1<sup>st</sup> die outcome and 2<sup>nd</sup> number in bracket represents 2<sup>nd</sup> die outcome

Example (a b): a is 1<sup>st</sup> die outcome and b is 2<sup>nd</sup> die outcome

[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),  
 (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),  
 (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),  
 (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),  
 (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)]  
 Total Possibilities = 36 [As  $6^2 = 36$ ]

**Problem 1:**

If 2 dice are thrown what is probability of

a) Getting same outcome on both

Total Possibilities = 36

Favourable to Getting same outcome on both dice  
 = 6 [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)]

Probability of getting same outcome on both dice =  $\frac{6}{36} = \frac{1}{6}$

b) Getting Product as perfect square

Total Possibilities = 36

Square of any natural number is called as perfect square 1, 4, 9, 16...all are perfect square. We need to find such pairs for which product is perfect square.

Favourable to getting Product as perfect square

= 8 [(1, 1), (1, 4), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5), (6, 6)]

Probability of getting Product as perfect square =  $\frac{8}{36} = \frac{2}{9}$

c) Getting Sum less than 6

Total Possibilities = 36

Favourable to getting sum less than 6

= 10 [(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)]  
 Probability of getting Sum less than 6 =  $\frac{10}{36} = \frac{5}{18}$

**Playing Cards Summary**

Total cards = 52 Cards

Red Cards = 26 Cards (13 Heart + 13 Diamond)

Black Cards = 26 Cards (13 Club + 13 Spade)

Face Cards: (3 in each set which become 12)

NonFace Cards =  $52 - 12 = 40$

**Problem 1:**

If 1 card is drawn at random, what is probability of

a) Getting black colour card

Total Possibilities = 52

Favourable to Getting black colour card

= 26 [13 Spade and 13 Club]

Probability of Getting black colour card =  $\frac{26}{52} = \frac{1}{2}$

b) It is an ace card

Total Possibilities = 52

Favourable to Getting an ace card = 4 [1 Club, 1 Spade, 1 Heart and 1 Diamond]

Probability of Getting an ace card =  $\frac{4}{52} = \frac{1}{13}$

c) It is a non-face card

Total Possibilities = 52

Favourable to get non-face card = 40 [10 Club, 10 Spade, 10 Heart and 10 Diamond]

Probability of Getting non face card =  $\frac{40}{52} = \frac{10}{13}$

**Problem 2:**

If 1 card is drawn at random, what is probability of

a) It is red queen

Total Possibilities = 52

Favourable to Getting red queen = 2 [Heart queen, Diamond queen]

$$\text{Probability of Getting red queen} = \frac{2}{52} = \frac{1}{26}$$

b) It is either queen or king

Total Possibilities = 52

Favourable to Getting either queen or king = 8 [4 King, 4 Queens]

$$\text{Probability of Getting either queen or king} = \frac{8}{52} = \frac{2}{13}$$

c) It is neither queen nor king

Total Possibilities = 52

Favourable to Getting neither queen nor king =  $52 - 8 = 44$  [Consider all card other than King and queen]

$$\text{Probability of Getting neither queen nor king} = \frac{44}{52} = \frac{11}{13}$$

**Profit & Loss****Real time example of Profit & Loss**

Let us assume as student if we study properly 8-10 hours per day, is it going to benefit us???

Yes definitely This is nothing but our profit.

If we don't study at all, is it going to benefit us???

Obviously No. This is our Loss

Similarly

If shopkeeper purchase any item at certain price and sell it at more price then it is Profit.

If shopkeeper sell same at lesser price then it is Loss.

**Terminologies of Profit & Loss:**

Rohit purchased bike at 40,000 Rs and sold it at 50,000 Rs What is Profit?

**Cost Price (CP):** The amount at which shopkeeper buys any Product  
(40,000 Rs in this case)

**Selling Price (SP):** The amount at which shopkeeper sells any product  
(50,000 Rs in this case)

**Profit = Selling Price – Cost Price**

$$= 50000 - 40000$$

**Profit = 10000 Rs**

**Loss = Cost Price – Selling Price**

$$\text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$\text{Loss \%} = \frac{\text{Loss}}{\text{CP}} \times 100$$

Always remember CP is base for Profit% as well as Loss%

| Sr No | CP   | SP   | Profit/Loss | Why     | Amount of Profit / Loss |
|-------|------|------|-------------|---------|-------------------------|
| 1     | 100  | 120  | Profit      | SP > CP | 20                      |
| 2     | 200  | 180  | Loss        | CP > SP | 20                      |
| 3     | 800  | 500  | Loss        | CP > SP | 300                     |
| 4     | 1000 | 1500 | Profit      | SP > CP | 500                     |

CP = Cost Price

SP = Selling Price

**Problem 1:**

Reema purchased mobile at 10,000 Rs and she sold it at 10% profit. What is Selling Price

**Solution**

CP = 10000 Rs

Profit = 10%

Let us assume CP as 100 Rs

To get 10% Profit, we need to sell it at  $100 + 10 = 110$  Rs

CP                      SP

100 ----- 110

10000-----x

Cross multiply

$100 \times x = 110 \times 10000$

$$x = \frac{110 \times 10000}{100} = \frac{110 \times 100}{1} = 11000$$

**Selling price is 11000 Rs**

**Problem 2:**

Ritika purchased Television at certain price and sold it at 20 Loss. If Ritika has sold it at 10,000 Rs then at what price she purchased it?

**Solution**

Loss = 20%

SP = 10000 Rs

CP = ?



$$SP = 10000 \text{ Rs}$$

$$\text{Profit/Loss\%} = ?$$

As  $SP > CP$ , it is Profit

$$\text{Profit} = SP - CP$$

$$= 10000 - 9000$$

$$\text{Profit} = 1000 \text{ Rs}$$

$$\text{Profit \%} = \frac{\text{Profit}}{CP} \times 100$$

$$\text{Profit \%} = \frac{1000}{9000} \times 100 = \frac{1}{9} \times 100 = \frac{100}{9} = 11.11\%$$

**Profit is 11.11%**



**Ratio & Proportion****Real time example of Ratio**

Let us try to understand real time meaning and usage of ratio.

Assume you have to prepare 1 cup Tea.

Most of you will be using quantity as 1 spoon tea powder, 2 spoons sugar

We can say that ratio of quantity of tea powder to quantity of sugar for 1 cup is 1 : 2 [1 as to 2]

Now if you have to prepare 10 cups of tea. How much quantity of Tea powder will be needed?

$10 \times 1 = 10$  spoons [As 1 cup needs 1 spoon tea powder]

How much quantity of sugar will be needed?

$10 \times 2 = 20$  spoons [As 1 cup needs 2 spoon Sugar]

Ratio is found by dividing 1st quantity by 2nd quantity

$10/20 = 1/2 = 1 : 2$

So, even though you prepare 1 cup/10 cup/100 cup of tea ratio of tea powder as to sugar powder remains same and will be 1 : 2

**Finding Ratio in Simplest form**

1) 22 : 66

$22/66 = 11/33 = 1/3 = 1 : 3$

2) 500 gm : 800 gm

$500/800 = 5/8 = 5 : 8$

3) 500 gm : 4 Kg

While converting/finding ratio, unit of both quantities should be same

Let us convert Kg to Gm

$4 \text{ Kg} = 4 \times 1000 = 4000 \text{ gm}$

$500 : 8000 \Rightarrow 500/4000 = 1/8 = 1 : 8$

4) 200m : 5 Km

Let us convert km into meter

$5 \text{ Km} = 5 \times 1000 = 5000 \text{ m}$

$$200 : 5000 = 200/5000 = 1/25 = 1 : 25$$

$$5) 3 \text{ hours} : 1 \text{ day}$$

Let us convert day into hours

$$1 \text{ day} = 24 \text{ hours}$$

$$3 : 24 = 1 : 8$$

$$6) 6 \text{ months} : 1 \frac{1}{3} \text{ years}$$

Let us convert years into months.

$$\frac{4}{3} \times 12 = 16 \text{ months}$$

$$6 : 16 = 3 : 8$$

$$7) \frac{1}{2} : \frac{1}{3}$$

Ratio means showing relationship between 2 quantities. By looking at ratio in fractions, we won't be able to visualize relationship between quantities clearly. So let us make it natural number.

Find LCM of denominators and multiply by same to both fractions.

LCM of 2 and 3 is 6

$$6 \times \frac{1}{2} : 6 \times \frac{1}{3}$$

$$3 : 2$$

### **Finding value based on Given value**

$$1) x : y = 3 : 4 \text{ Find value of } (4x + 5y) : (5x - 2y)$$

Put  $x=3$  and  $y=4$

$$\begin{aligned} (4x + 5y) : (5x - 2y) &= ((4 \times 3) + (5 \times 4)) : ((5 \times 3) - (2 \times 4)) \\ &= (12 + 20) : (15 - 8) \end{aligned}$$

**Word Problems**

1) The ratio between two quantities is 3:4

If first is 810 Rs find second?

**Solution**

Ratio of 2 quantities = 1st Quantity/2nd Quantity

$$3/4 = 810/x$$

Cross multiply

$$3 \times x = 810 \times 4$$

$$x = 810 \times 4 / 3 = \mathbf{1080 \text{ Rs}}$$

2) Two numbers are in Ratio 10:11 Their sum is 168 Find the numbers

**Solution**

Ratio of 2 numbers is 10 : 11

There will be some common multiple x by which if we multiply by 10 and 11, we get both numbers

$$\text{1st number} = 10 \times x = 10x$$

$$\text{2nd number} = 11 \times x = 11x$$

Their sum is 168

$$10x + 11x = 168$$

$$21x = 168$$

$$x = 168/21 = 8$$

$$\text{1st number} = 10x = 10 \times 8 = \mathbf{80}$$

$$\text{2nd number} = 11x = 11 \times 8 = \mathbf{88}$$

3) Two numbers are in ratio 5 7 Their difference is 10. Find the numbers.

**Solution:**

Ratio is 5 : 7

Common Multiple x

1st number 5x and 2nd number 7x

5x is smaller and 7x is bigger

Their difference is 10

$$7x - 5x = 10$$

$$2x = 10$$

$$x = 10/2 = 5$$

$$\text{1st number} = 5x = 5 \times 5 = \mathbf{25}$$

$$\text{2nd number} = 7x = 7 \times 5 = \mathbf{35}$$

4) Ratio between two numbers is  $3 : 4$  Their HCF is 15. Find numbers.

**Solution:**

Let us assume common multiple  $x$

Numbers are  $3x$  and  $4x$

$$3x = 3 \times x \quad 4x = 2 \times 2 \times x$$

Looking at this we can say HCF is  $x$

However HCF is given as 15

$$x = 15$$

$$\text{1st number} = 3x = 3 \times 15 = \mathbf{45}$$

$$\text{2nd number} = 4x = 4 \times 15 = \mathbf{60}$$

5) Ratio between two numbers is  $4 : 7$  Their LCM is 168. Find numbers.

**Solution:**

Let us assume common multiple  $x$

Numbers are  $4x$  and  $7x$

$$4x = 2 \times 2 \times x \quad 7x = 7 \times x$$

Looking at this we can say LCM is

$$2 \times 2 \times 7 \times x = 28x$$

However LCM is given as 168

$$28x = 168$$

$$x = 168/28 = 6$$

$$\text{1st number} = 4x = 4 \times 6 = \mathbf{24}$$

$$\text{2nd number} = 7x = 7 \times 6 = \mathbf{42}$$

6) Divide 260 Rs among A, B and C in ratio of  $1/2 : 1/3 : 1/4$

**Solution:**

We need to convert ratio into simplest form first

LCM of 2, 3, 4 is 12

Multiply all fractions by 12

$$12 \times \frac{1}{2} : 12 \times \frac{1}{3} : 12 \times \frac{1}{4}$$

$$6 : 4 : 3$$

Let us assume common multiple as  $x$ .

It will be distributed in A, B, C as  $6x, 4x, 3x$

$$6x + 4x + 3x = 260$$

$$13x = 260 \text{ Rs}$$

$$x = 260/13 = 20$$

$$\text{A will get } 6x = 6 \times 20 = \mathbf{120 \text{ Rs}}$$

$$\text{B will get } 4x = 4 \times 20 = \mathbf{80 \text{ Rs}}$$

$$\text{C will get } 3x = 3 \times 20 = \mathbf{60 \text{ Rs}}$$

7) Ratio between prices of scooter and refrigerator is  $4 : 1$ . If scooter costs 45000 Rs more than refrigerator, find price of refrigerator

**Solution:**

$$\text{Scooter : Refrigerator price} = 4 : 1$$

Let us assume common multiple as  $x$

$$\text{Scooter price} = 4x$$

$$\text{Refrigerator price} = x$$

$$\text{Scooter price} = \text{Refrigerator price} + 45000$$

$$4x = x + 45000$$

$$3x = 45000$$

$$x = 45000/3 = 15000$$

$$\text{Refrigerator price} = x = \mathbf{15000 \text{ Rs}}$$

**Proportion concept**

In simple words proportion means checking 2 ratios are equal or not

Let us take 2 ratios

10 15 and 8 12

10 15

8 12

10/15 = 2 3

8/12 = 2 :3

By looking at both ratio we can say both are equal

We can say **10, 15, 8, 12 are in proportion.**

If both were not same we can say that 10, 15, 8, 12 not in proportion

Let us assume 4 quantities a, b, c, d

a. b : c : d

$a/b = c/d$

$a \times d = b \times c$

Product of extremes = Product of means

1st X 4th = 2nd X 3rd

**Proportion Check**

Check whether following quantities are in proportion or not

1) 10, 20, 40, 75

**Solution:**

Check Product of extremes (1st X 4th) and Product of Means(2nd X 3rd) If it matches then numebrs are n proportion Else it is not.

Product of Extremes

$10 \times 75 = 750$

Product of Means

$20 \times 40 = 800$

750 not equal 800

**10, 20, 40, 75 not in proportion**

**Find missing number**

Below 4 number/quantities are in proportion Find value in place of question mark

For solving let us assume missing number as x

1) 5, 9, 45, ?

**Solution:**

$$5 \times x = 9 \times 45$$

$$x = 9 \times 45 / 5 = 81$$

**Missing number is 81**

2) 4 hour 40 minutes, 1 hour 10 minutes 16 hour ?

Let us convert all into minutes

$$4 \text{ hour } 40 \text{ minutes} = (60 \times 4) + 40 = 280 \text{ minutes}$$

$$1 \text{ hour } 10 \text{ minutes} = (60 \times 1) + 10 = 70 \text{ minutes}$$

$$16 \text{ hour} = 60 \times 16 = 960 \text{ minutes}$$

$$280, 70, 960, x$$

$$280 \times x = 70 \times 960$$

$$x = 70 \times 960 / 280$$

$$x = 240 = 4 \text{ hours}$$

**Missing number is 4 hours**

**Continued Proportion Concept**

If Middle number<sup>2</sup> = 1st number X 3rd number then 3 numbers are in continued proportion

If Middle number<sup>2</sup> not equal to 1st number X 3rd number then 3 numbers are not in continued proportion.

1) 8, 16, 32

$$16^2 \quad 8 \times 32$$

$$256 = 256$$

**8, 16, 32 are in continued proportion**

2) 10, 9, 8

$$9^2 \quad 10 \times 8$$

81 not equal to 80

**10, 9, 8 are not in continued proportion**

3) 2x, 4x, 8x

$$(4x)^2 \quad 2x \times 8x$$

$$16x^2 = 16x^2$$

**2x, 4x, 8x are in Continued proportion**

### **Continued Proportion - Find missing number**

Below 3 numbers are in continued proportion

Find missing number in place of question mark

Let us assume missing number as x

1) 2, 10, ?

$$2 \times x = 10^2$$

$$x = 100/2 = 50$$

**Missing number is 50**

2) 0.5, 1.5, ?

$$0.5 \times x = 1.5^2$$

$$x = 2.25/0.5 = 225/50 = 9/2 = 4.5$$

**Missing number is 4.5**

3) 2x, 8x, ?

Here x is already mentioned so let us take missing number as a

$$2x \times a = (8x)^2$$

$$a = 64x^2/2x = 32x$$

**Missing number is 32x**



**Rounding Off****Real time usage of Rounding Off Concept:**

Knowingly or Unknowingly we Use rounding off in day to day life  
We will try to understand concept of Rounding Off with very simple example

Assume you are in your house. One of your friend comes to meet you.  
He/she asks for you and you are already doing some work. What you will say to him/her?

Wait I will come in 2 minutes

Or

Wait I will come in 10 minutes

Or

Wait I will come in half hour

Or

Can you come after 1 hour so that I will be free by that time

Do you remember anytime you are responding like

I will come in 7 minutes

Or

I will come in 47 seconds

Definitely No

We and our friend always comfortable in understanding Rounded numbers  
(like 10, 100, 1000 etc.)

This is reason we should learn and capable of rounding off numbers

We are going to see SIMPLE method which will help us to round off any number to nearest ten(10), hundred (100), thousand(1000), ten thousand (10000)

**Rounding off to Nearest Ten:**

Step 1: How many zero in Ten?

1 Zero. So wrote 1 zero from right side

Step 2: As 10 is having 1 zero, check 1st digit from right side

Step 3: If that number is greater than or equal to 5 then increase remaining part by 1

Or

If that number is less than 5 then keep remaining part as is

**Problem 1:**

73  
↓  
Step 1      0  
Step 2      3  
Step 3      3 is less than 5  
              Remaining part 7  
              70

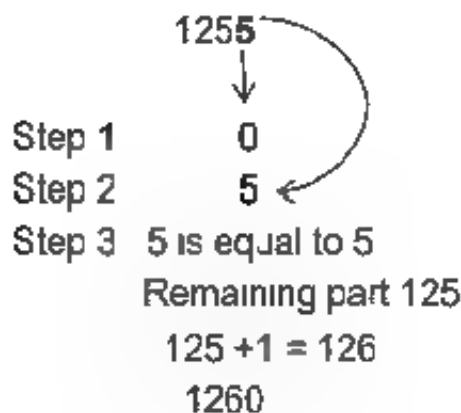
**Rounded number is 70**

**Problem 2:**

87  
↓  
Step 1      0  
Step 2      7  
Step 3      7 is greater than 5  
              Remaining part 8  
               $8 + 1 = 9$   
              90

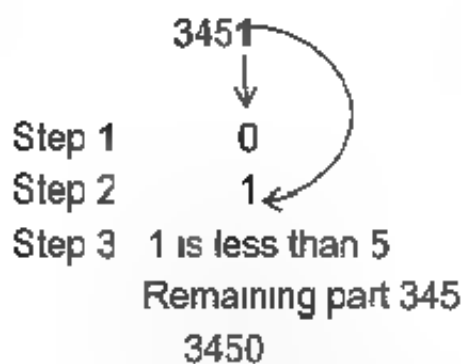
**Rounded number is 90**

**Problem 3:**



**Rounded number is 1260**

**Problem 4:**



**Rounded number is 3450**

**Rounding Off - Nearest Hundred**

Step 1: How many zero in Hundred?

2 Zero. So write 2 zero from right side

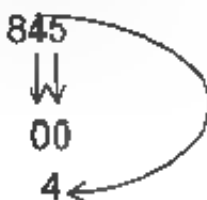
Step 2: As 100 is having 2 zero, check 2nd digit from right side

Step 3: If that number is greater than or equal to 5 then increase remaining part by 1

Or

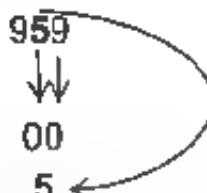
If that number is less than 5 then keep remaining part as is

## Problem 1:

$845$   
  
 Step 1      00  
 Step 2      4  
 Step 3    4 is less than 5  
             Remaining part 8  
             800

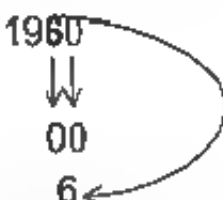
**Rounded number is 800**

## Problem 2:

$959$   
  
 Step 1      00  
 Step 2      5  
 Step 3    5 is equal to 5  
             Remaining part 9  
              $9 + 1 = 10$   
             1000

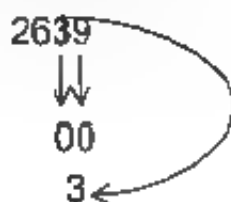
**Rounded number is 1000**

## Problem 3:

$1960$   
  
 Step 1      00  
 Step 2      6  
 Step 3    6 is greater than 5  
             Remaining part 19  
              $19 + 1 = 20$   
             2000

**Rounded number is 2000**

**Problem 4:**


  
 Step 1      00
   
 Step 2      3
   
 Step 3    3 is less than 5
   
             Remaining part 26
   
             2600

**Rounded number is 2600**

**Rounding Off – Nearest Thousand**

Step 1: How many zero in Thousand?

3 Zero. So write 3 zero from right side

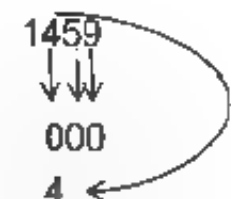
Step 2: As 1000 is having 3 zero, check 3rd digit from right side

Step 3: If that number is greater than or equal to 5 then increase remaining part by 1

Or

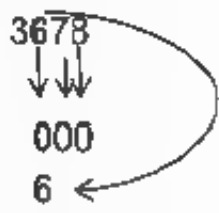
If that number is less than 5 then keep remaining part as is

**Problem 1:**


  
 Step 1      000
   
 Step 2      4
   
 Step 3    4 is less than 5
   
             Remaining part 1
   
             1000

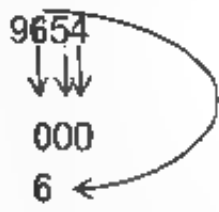
**Rounded number is 1000**

**Problem 2:**


  
 Step 1      000
   
 Step 2      6
   
 Step 3    6 is greater than 5
   
           Remaining part  $3+1 = 4$ 
  
           4000

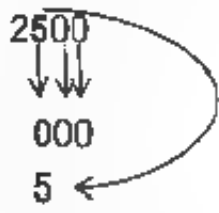
**Rounded number is 4000**

**Problem 3:**


  
 Step 1      000
   
 Step 2      6
   
 Step 3    6 is greater than 5
   
           Remaining part  $9+1 = 10$ 
  
           10000

**Rounded number is 10000**

**Problem 4:**


  
 Step 1      000
   
 Step 2      5
   
 Step 3    5 is equal to 5
   
           Remaining part  $2+1 = 3$ 
  
           3000

**Rounded number is 3000**

**Rounding Off – Nearest Ten Thousand**

Step 1: How many zero in Ten Thousand?

4 Zero. So write 4 zero from right side

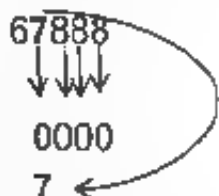
Step 2: As 10000 is having 4 zero, check 4th digit from right side

Step 3: If that number is greater than or equal to 5 then increase remaining part by 1

Or

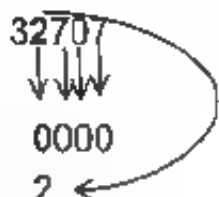
If that number is less than 5 then keep remaining part as is

**Problem 1:**

  
 Step 1      0000  
 Step 2      7  
 Step 3    7 is greater than 5  
             Remaining part 6  
              $6+1 = 7$

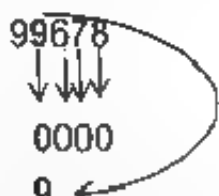
**Rounded number is 70000**

**Problem 2**

  
 Step 1      0000  
 Step 2      2  
 Step 3    2 is less than 5  
             Remaining part 3  
             30000

**Rounded number is 30000**

**Problem 3**

  
 Step 1      0000  
 Step 2      9  
 Step 3    9 is greater than 5  
             Remaining part  $9+1 = 10$   
             100000

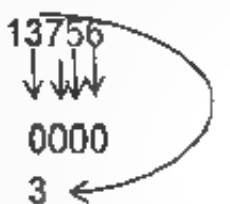
**Rounded number is 100000**

## Aptitude Made Simple

## Rounding Off

### Problem 4

13756  
↓ ↓ ↓ ↓  
Step 1      0000  
Step 2      3  
Step 3      3 is less than 5  
            Remaining part 1  
            10000



**Rounded number is 10000**

:



## Trigonometry

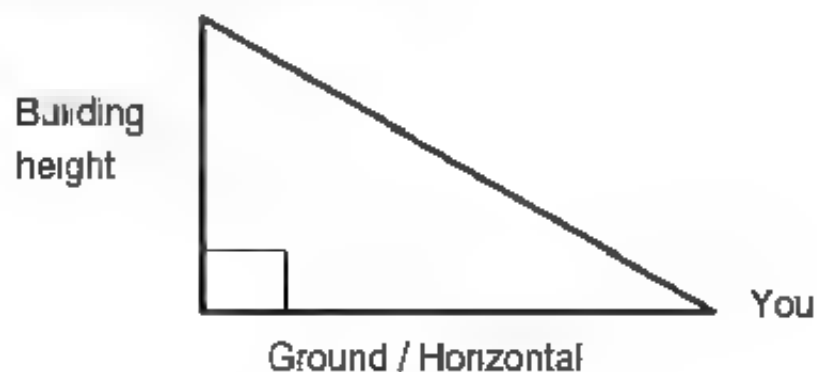
### Real time example of Trigonometry

Do you use  $\sin A$ ,  $\cos A$  in your day to day life?? Then why do we put so much effort in learning trigonometry?

Assume you are standing in front of tall building. How will you measure its height?

Using measuring tape???

Obviously NOT!!! Solution is trigonometry



You know

- 1) Distance from you to building
- 2) Devices which help you to set angle so that you see top of building

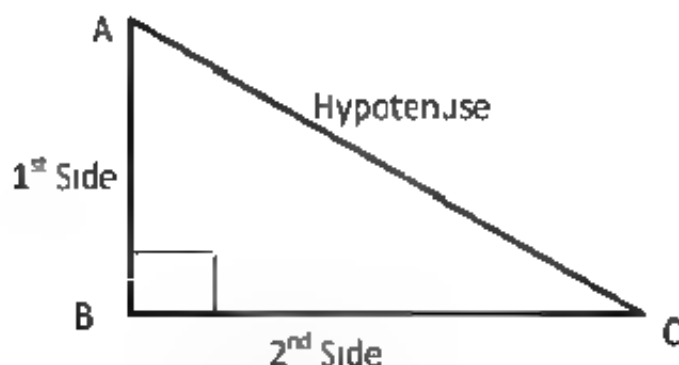
Using these 2 things and trigonometric ratio you can find height of building. Don't worry now at this moment.

Right angled triangle

If any 1 angle in triangle is  $90^\circ$  then it is called as Right angled triangle.

We know sum of all angles of any triangle is  $180^\circ$ . So if 1 angle is  $90^\circ$ , sum of other 2 angles is  $90^\circ$ . So  $90^\circ$  angle is biggest. Opposite side to biggest ~~side~~ which is called as Hypotenuse.

angle



$$\text{Hypotenuse}^2 = 1^{\text{st}} \text{ Side}^2 + 2^{\text{nd}} \text{ Side}^2$$

$$AC^2 = AB^2 + BC^2$$

All Trigonometric ratios can be found out if given triangle is Right angled triangle.

### Right angled triangle – Find hypotenuse

#### Problem 1:

In Triangle ABC, Angle ABC =  $90^\circ$ , AB = 8 cm and BC = 6 cm

#### Solution :

$$\text{Hypotenuse}^2 = 1^{\text{st}} \text{ Side}^2 + 2^{\text{nd}} \text{ Side}^2$$

$$AC^2 = AB^2 + BC^2$$

$$= 8^2 + 6^2$$

$$AC^2 = 64 + 36 = 100$$

$$AC = \sqrt{100}$$

$$\mathbf{AC = 10 \text{ cm}}$$

#### Problem 2:

In Triangle PQR, Angle PQR =  $90^\circ$ , PQ = 3 cm and QR = 4 cm

#### Solution :

$$\text{Hypotenuse}^2 = 1^{\text{st}} \text{ Side}^2 + 2^{\text{nd}} \text{ Side}^2$$

$$PR^2 = PQ^2 + QR^2$$

$$= 3^2 + 4^2$$

$$PR^2 = 9 + 16 = 25$$

$$PR = \sqrt{25}$$

$$\mathbf{PR = 5 \text{ cm}}$$

**Right angled triangle – Find Missing side****Problem 1:**

In Triangle ABC, Angle ABC =  $90^\circ$ , AC = 10 cm and BC = 6 cm, AB = ?

**Solution :**

$$\text{Hypotenuse}^2 = 1\text{st Side}^2 + 2\text{nd Side}^2$$

$$AC^2 = AB^2 + BC^2$$

$$10^2 = AB^2 + 6^2$$

$$100 = AB^2 + 36$$

$$100 - 36 = AB^2$$

$$AB^2 = 64$$

$$AB = \sqrt{64}$$

$$AB = 8 \text{ cm}$$

**Problem 2:**

In Triangle PQR, Angle PQR =  $90^\circ$ , QR = 10 cm and PR = 5 cm, PQ = ?

**Solution :**

$$\text{Hypotenuse}^2 = 1\text{st Side}^2 + 2\text{nd Side}^2$$

$$PR^2 = PQ^2 + QR^2$$

$$5^2 = PQ^2 + 4^2$$

$$25 = PQ^2 + 16$$

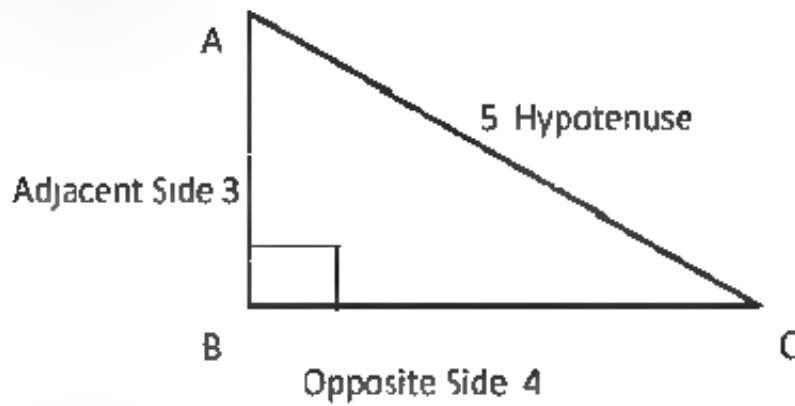
$$25 - 16 = PQ^2$$

$$PQ^2 = 9$$

$$PQ = \sqrt{9}$$

$$PQ = 3 \text{ cm}$$

## Trigonometry Ratios



$$\sin A = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{4}{5}$$

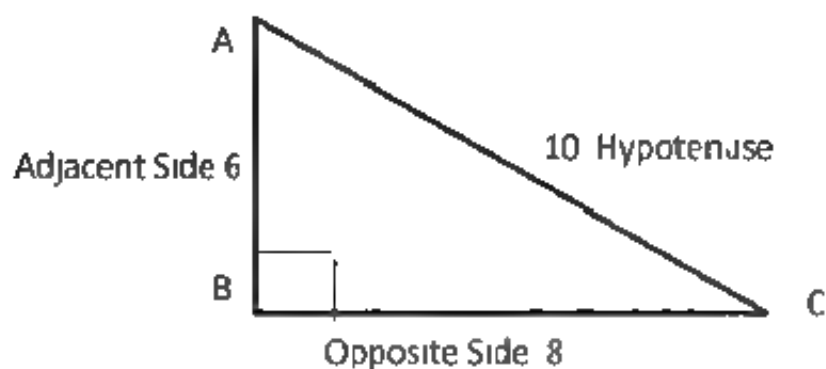
$$\cos A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{3}{5}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{4/5}{3/5} = \frac{4}{5} \times \frac{5}{3} = \frac{4}{3}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{4/3} = \frac{3}{4}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{4/5} = \frac{5}{4}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{3/5} = \frac{5}{3}$$



$$\sin A = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{8}{10} = \frac{4}{5}$$

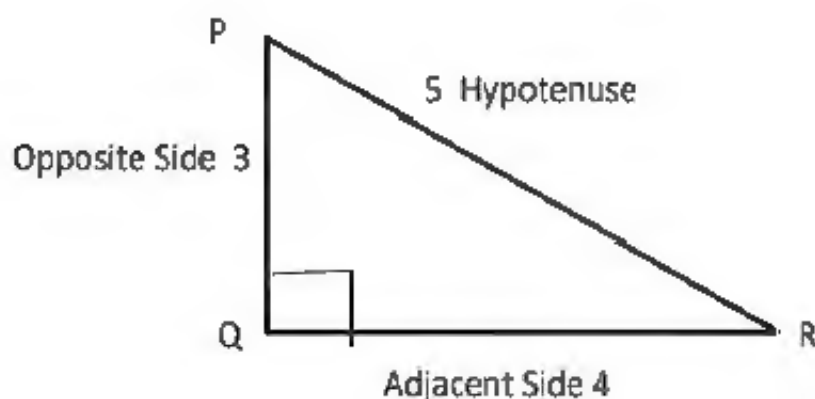
$$\cos A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{6}{10} = \frac{3}{5}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{4/5}{3/5} = \frac{4}{5} \times \frac{5}{3} = \frac{4}{3}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{4/3} = \frac{3}{4}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{4/5} = \frac{5}{4}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{3/5} = \frac{5}{3}$$



$$\sin R = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{3}{5}$$

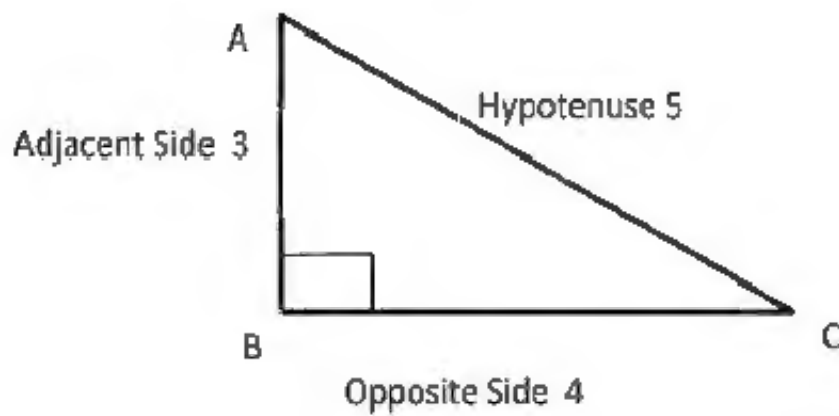
$$\cos R = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{4}{5}$$

$$\tan R = \frac{\sin R}{\cos R} = \frac{3/5}{4/5} = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$$

$$\cot R = \frac{1}{\tan R} = \frac{1}{3/4} = \frac{4}{3}$$

$$\operatorname{cosec} R = \frac{1}{\sin R} = \frac{1}{3/5} = \frac{5}{3}$$

$$\sec R = \frac{1}{\cos R} = \frac{1}{4/5} = \frac{5}{4}$$

Trigonometry Identities : 1<sup>st</sup> Identity

$$\sin A = \frac{4}{5}$$

$$\cos A = \frac{3}{5}$$

$$\sin^2 A + \cos^2 A$$

$$\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2$$

$$\frac{16}{25} + \frac{9}{25} = \frac{25}{25} = 1$$

$$\sin^2 A + \cos^2 A = 1$$

**Trigonometry Identities : 2<sup>nd</sup> Identity**

1<sup>st</sup> Identity is  $\sin^2 A + \cos^2 A = 1$

This will help us to find other 2 identities

$$1 + \tan^2 A$$

$$1 + \frac{\sin^2 A}{\cos^2 A} = \frac{\cos^2 A + \sin^2 A}{\cos^2 A} = \frac{1}{\cos^2 A} = \sec^2 A$$

$$1 + \tan^2 A = \sec^2 A$$

**Trigonometry Identities : 3<sup>rd</sup> Identity**

$$1 + \cot^2 A$$

$$1 + \frac{\cos^2 A}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\sin^2 A} = \frac{1}{\sin^2 A} = \operatorname{cosec}^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$



## Trigonometry Standard Angles Table

$0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $90^\circ$  are very much important.

If we find sin and cos we will get others easily

| Trick                             | $0^\circ$  | $30^\circ$   | $45^\circ$   | $60^\circ$   | $90^\circ$ |
|-----------------------------------|------------|--------------|--------------|--------------|------------|
| Write 0 to 4                      | 0          | 1            | 2            | 3            | 4          |
| Divide by 4                       | 0/4        | 1/4          | 2/4          | 3/4          | 4/4        |
| simplify                          | 0          | 1/4          | 1/2          | 3/4          | 1          |
| Take square root of all           | $\sqrt{0}$ | $\sqrt{1/4}$ | $\sqrt{1/2}$ | $\sqrt{3/4}$ | $\sqrt{1}$ |
| Sin                               | 0          | 1/2          | $1/\sqrt{2}$ | $\sqrt{3}/2$ | 1          |
| Cos (Copy sin from right to left) | 1          | $\sqrt{3}/2$ | $1/\sqrt{2}$ | 1/2          | 0          |
| Tan                               | 0          | $1/\sqrt{3}$ | 1            | $\sqrt{3}$   | ND         |
| Cot                               | ND         | $\sqrt{3}$   | 1            | $1/\sqrt{3}$ | 0          |
| Cosec                             | ND         | 2            | $\sqrt{2}$   | $2/\sqrt{3}$ | 1          |
| Sec                               | 1          | $2/\sqrt{3}$ | $\sqrt{2}$   | 2            | ND         |